Elias-Fano Encoding

Succinct representation of monotone integer sequences with search operations

Giulio Ermanno Pibiri
giulio.pibiri@di.unipi.it
Computer Science Department
University of Pisa

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Problem

Consider a sequence S[0,n) of n *positive* and *monotonically increasing integers*, i.e., S[i-1] ≤ S[i] for 1 ≤ i ≤ n-1, possibly repeated.

How to represent it as a *bit vector* in which each original integer is *self-delimited*, using as few as possible bits?

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Huge research corpora describing different space/time trade-offs.

- Elias gamma/delta [Salomon-2007]
- Variable Byte [Salomon-2007]
- Varint-G8IU [Stepanov et al.-2011]
- Simple-9/16 [Anh and Moffat 2005-2010]
- PForDelta (PFD) [Zukowski et al.-2006]
- OptPFD [Yan et al.-2009]
- Binary Interpolative Coding [Moffat and Stuiver-2000]

Inverted Indexes

1

Given a *textual collection* D, each document can be seen as a (multi-)set of terms. The set of terms occurring in D is the *lexicon* T.

For each term t in T we store in a list L_t the identifiers of the documents in which t appears.

The collection of all inverted lists $\{L_{t_1},\ldots,L_{t_T}\}$ is the inverted index.

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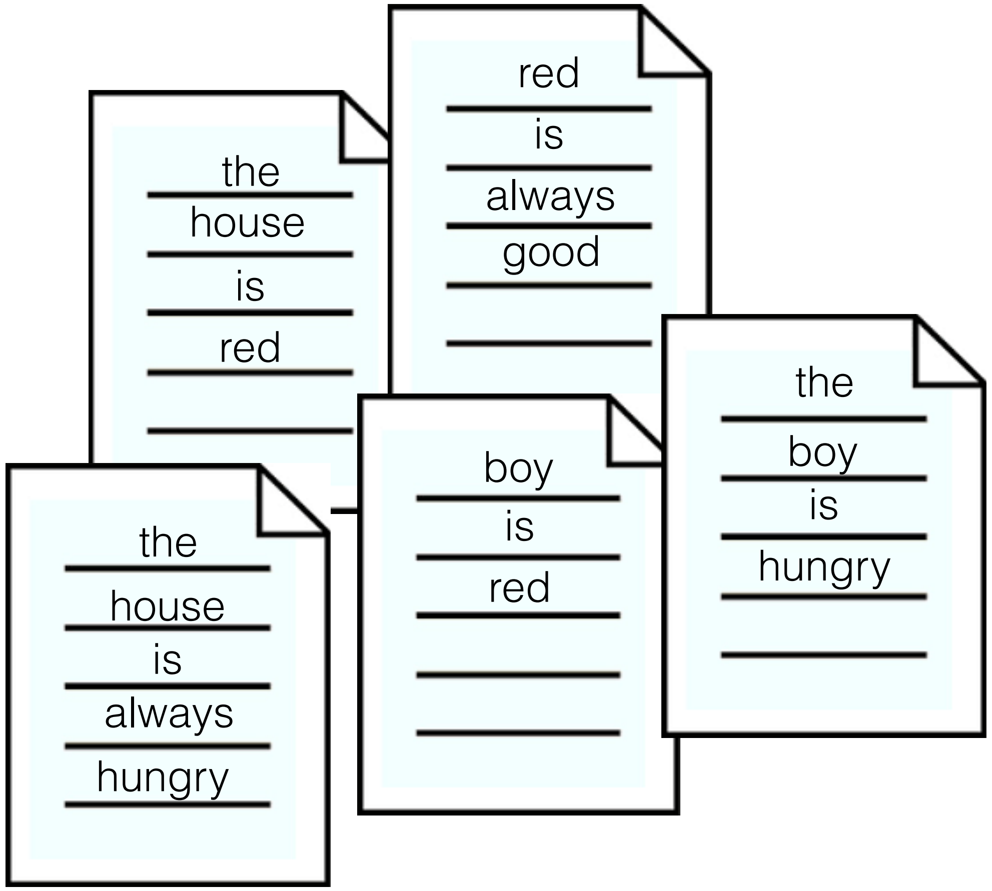

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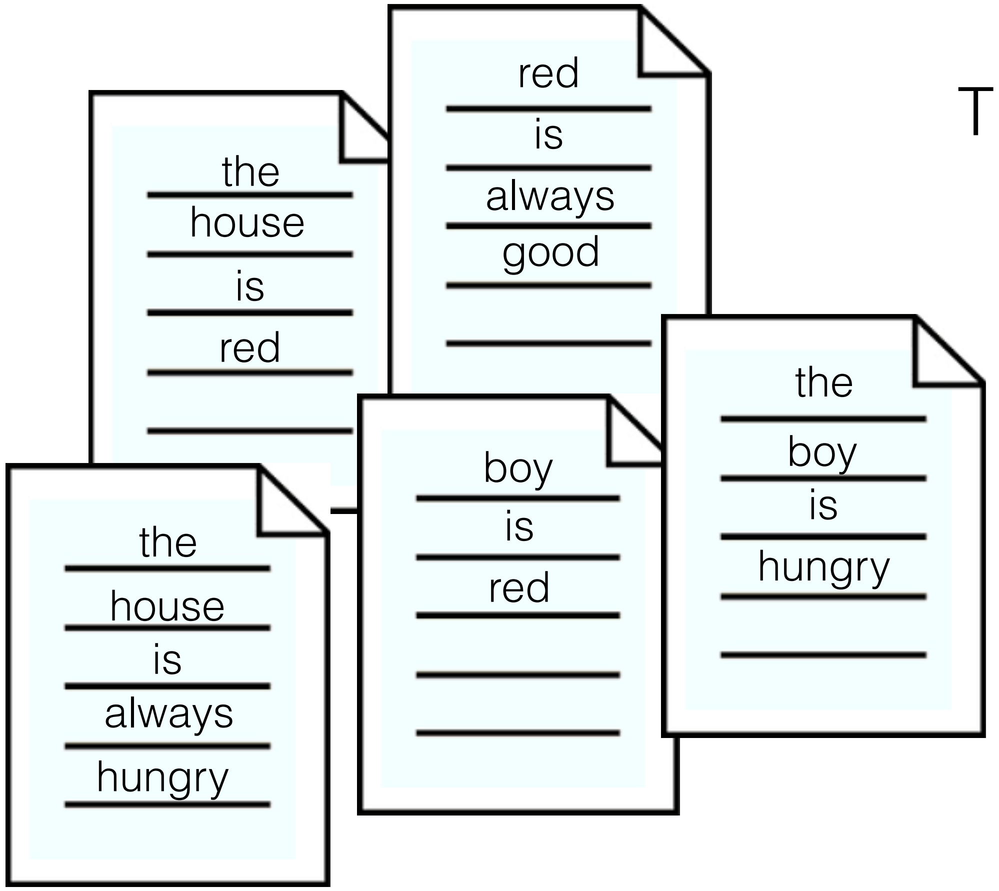

$$T = \{\overset{t_1}{\text{always}}, \overset{t_2}{\text{boy}}, \overset{t_3}{\text{good}}, \overset{t_4}{\text{house}}, \overset{t_5}{\text{hungry}}, \overset{t_6}{\text{is}}, \overset{t_7}{\text{red}}, \overset{t_8}{\text{the}}\}$$

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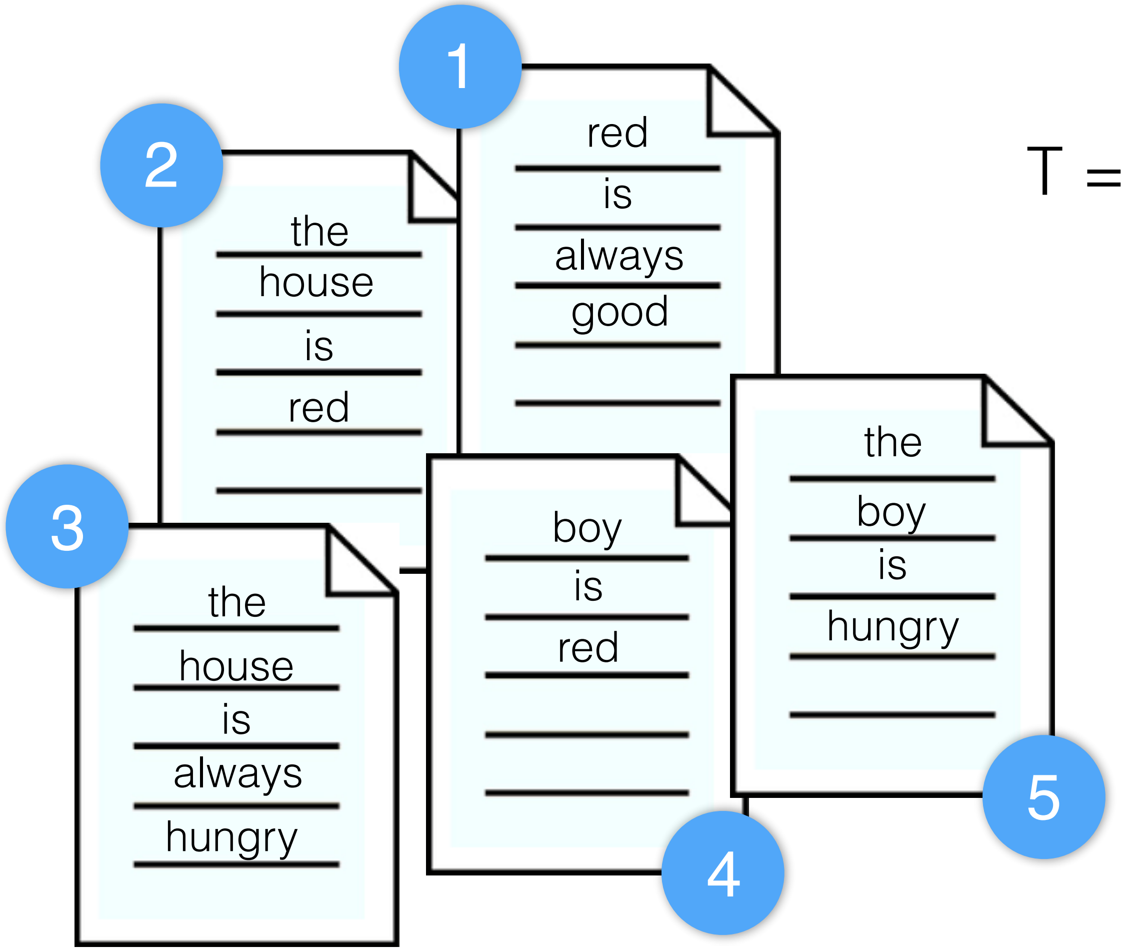

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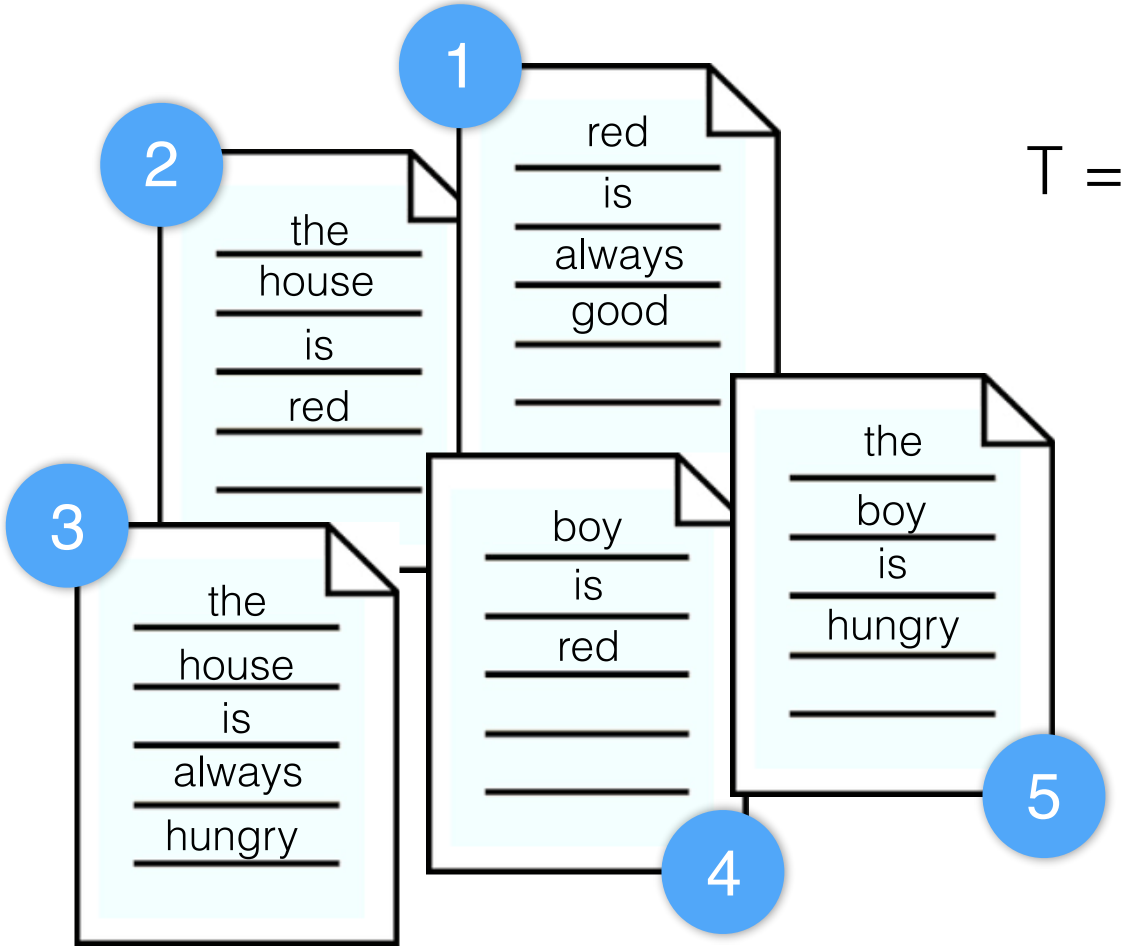

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Inverted Indexes owe their popularity to the *efficient resolution of queries*, such as: “return me all documents in which terms $\{t_1,\ldots,t_k\}$ occur”.

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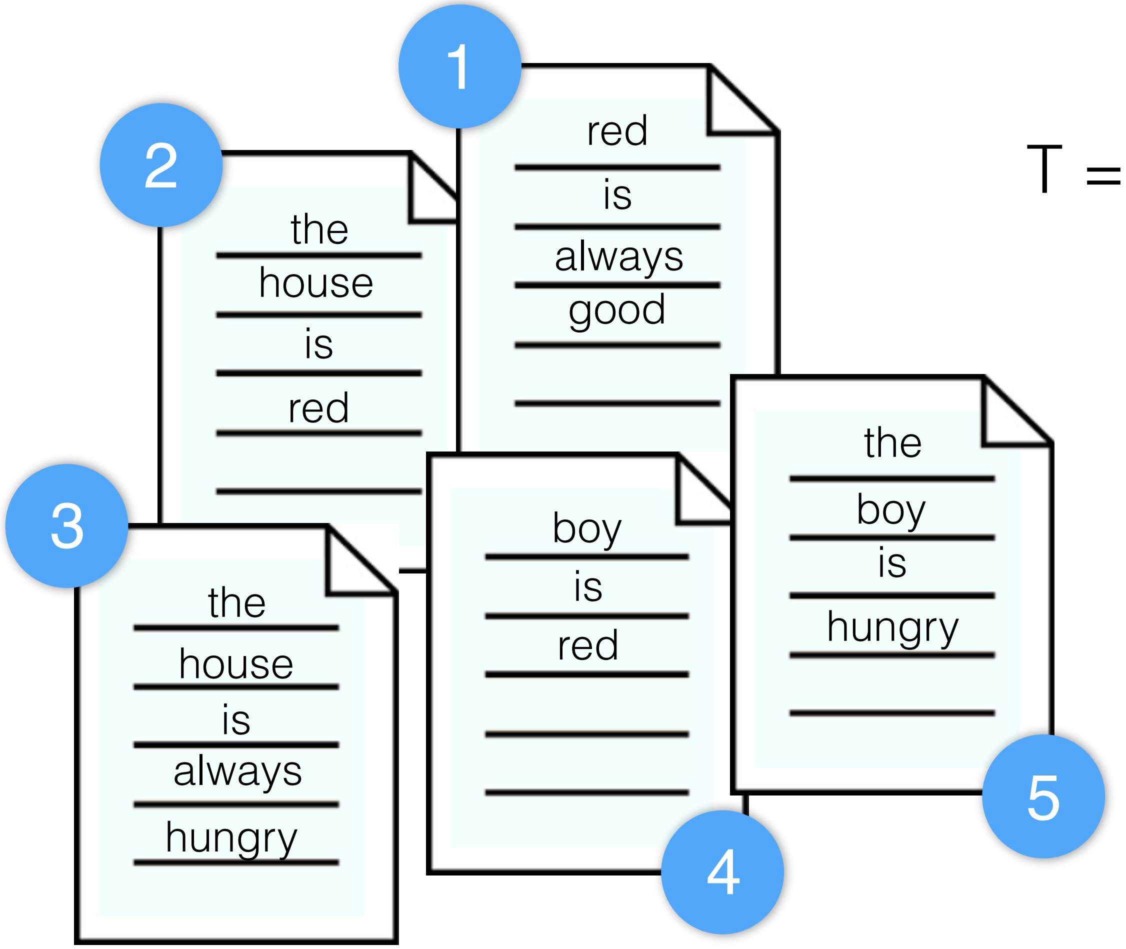

$$\begin{array}{cccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 \end{array}$$

$T = \{\text{always, boy, good, house, hungry, is, red, the}\}$

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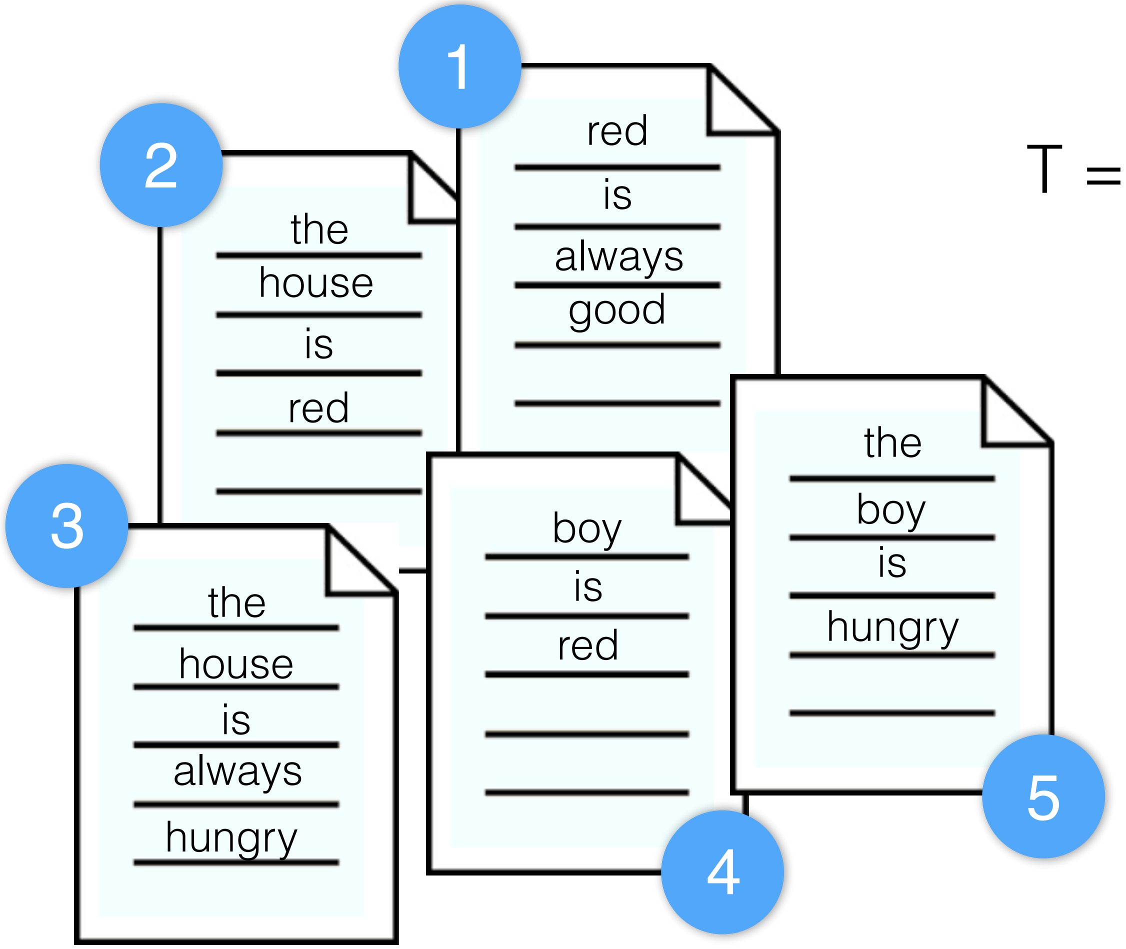

1
red
is
always
good

2
the
house
is
red

3
the
house
is
always
hungry

4
boy
is
red

5
the
boy
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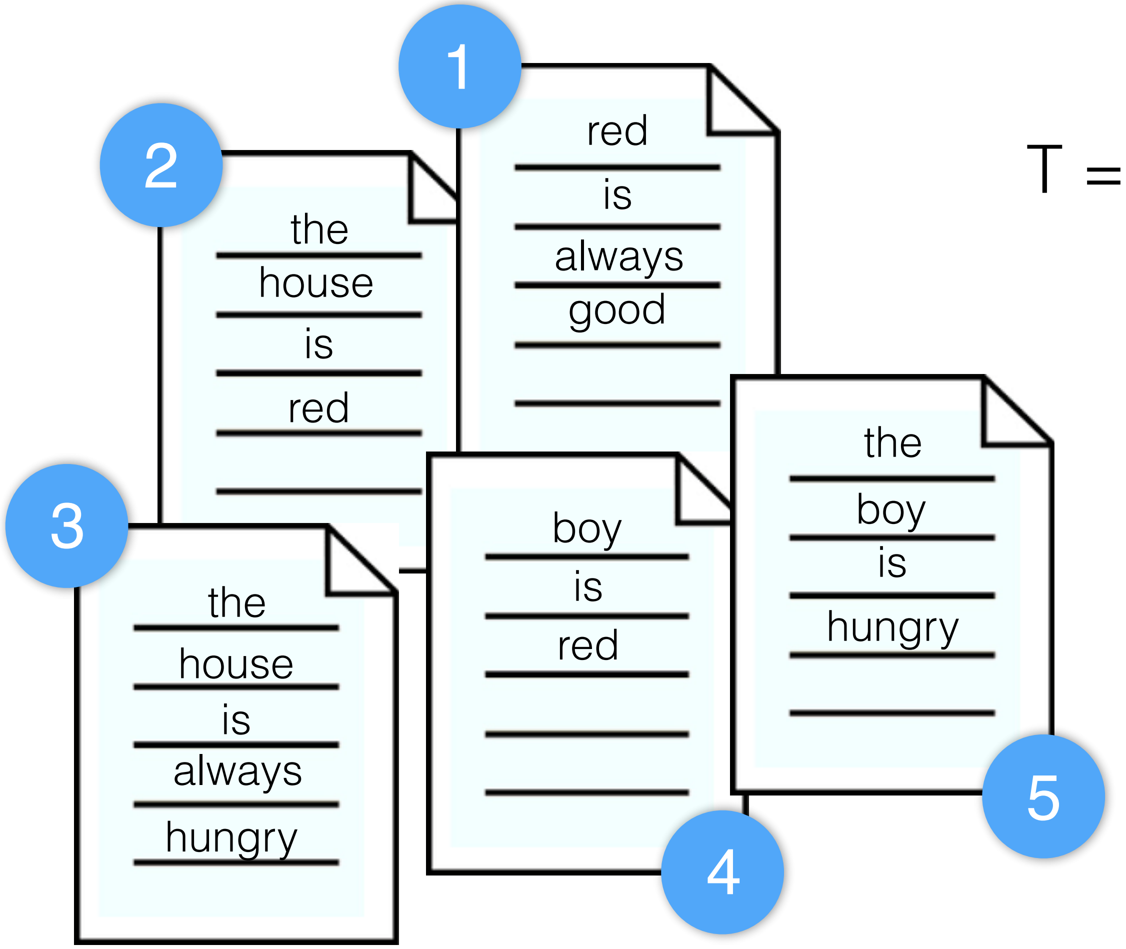

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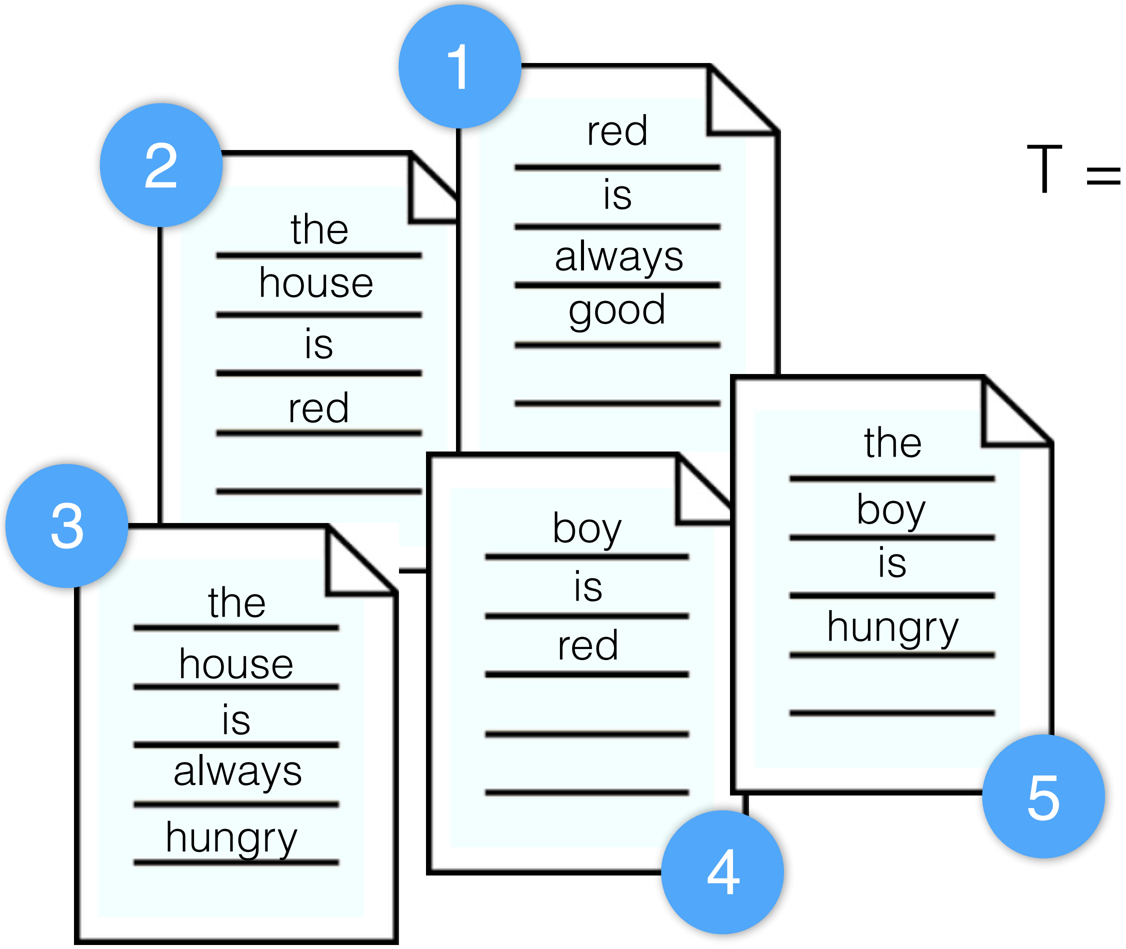

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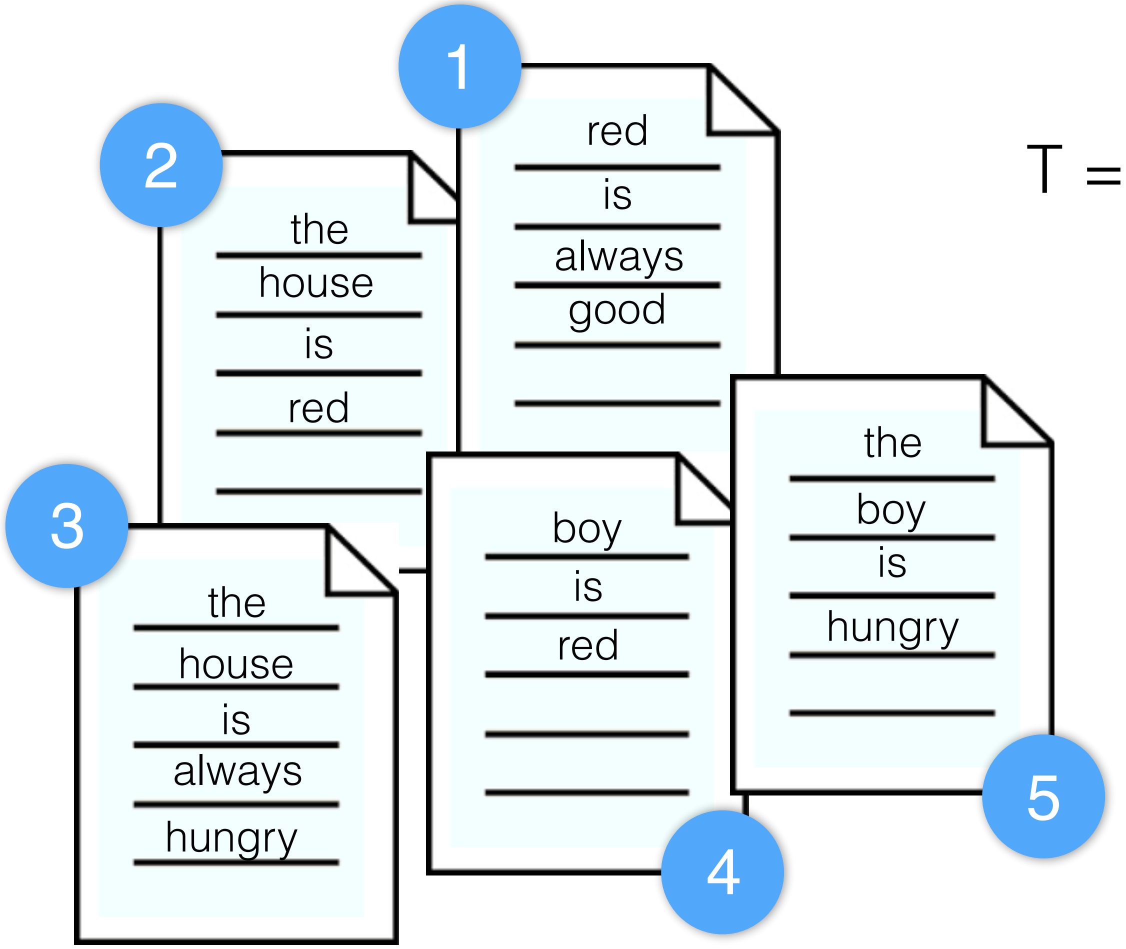

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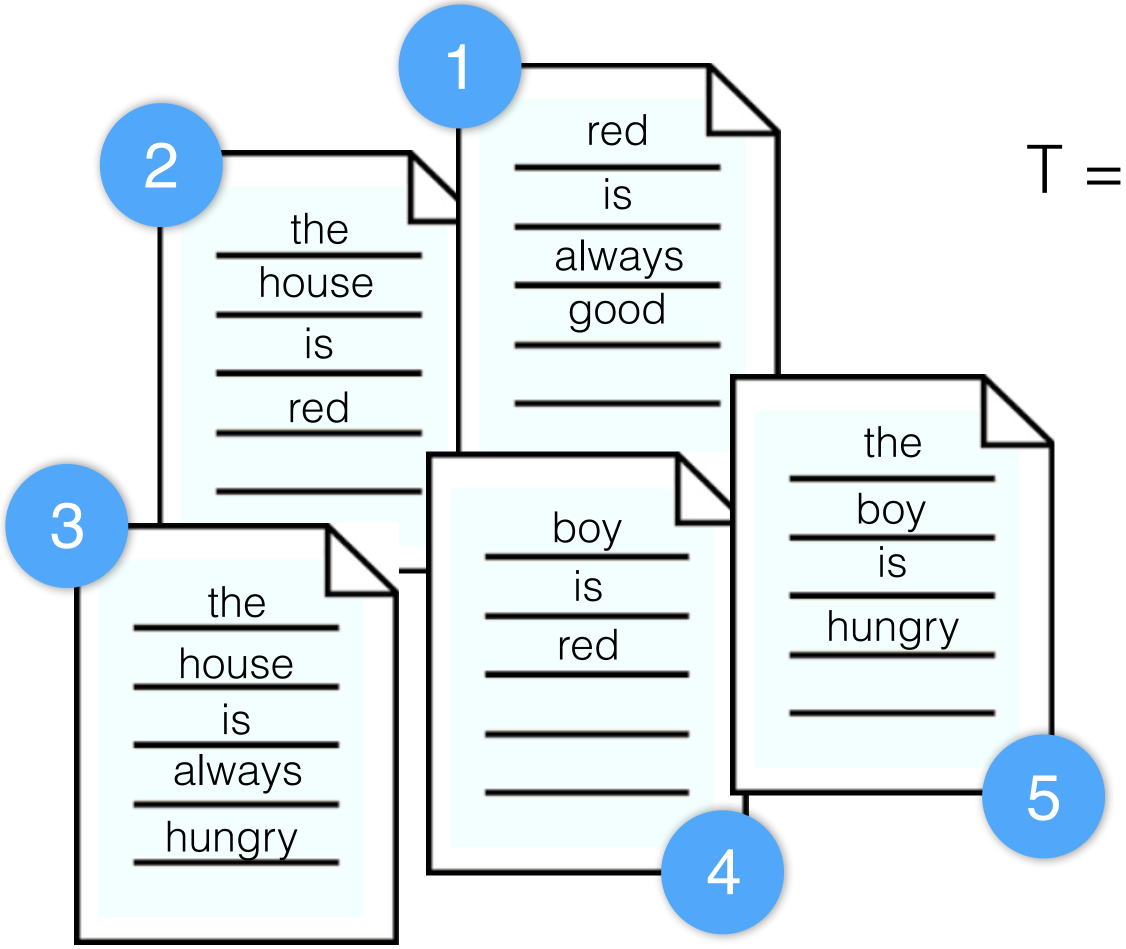

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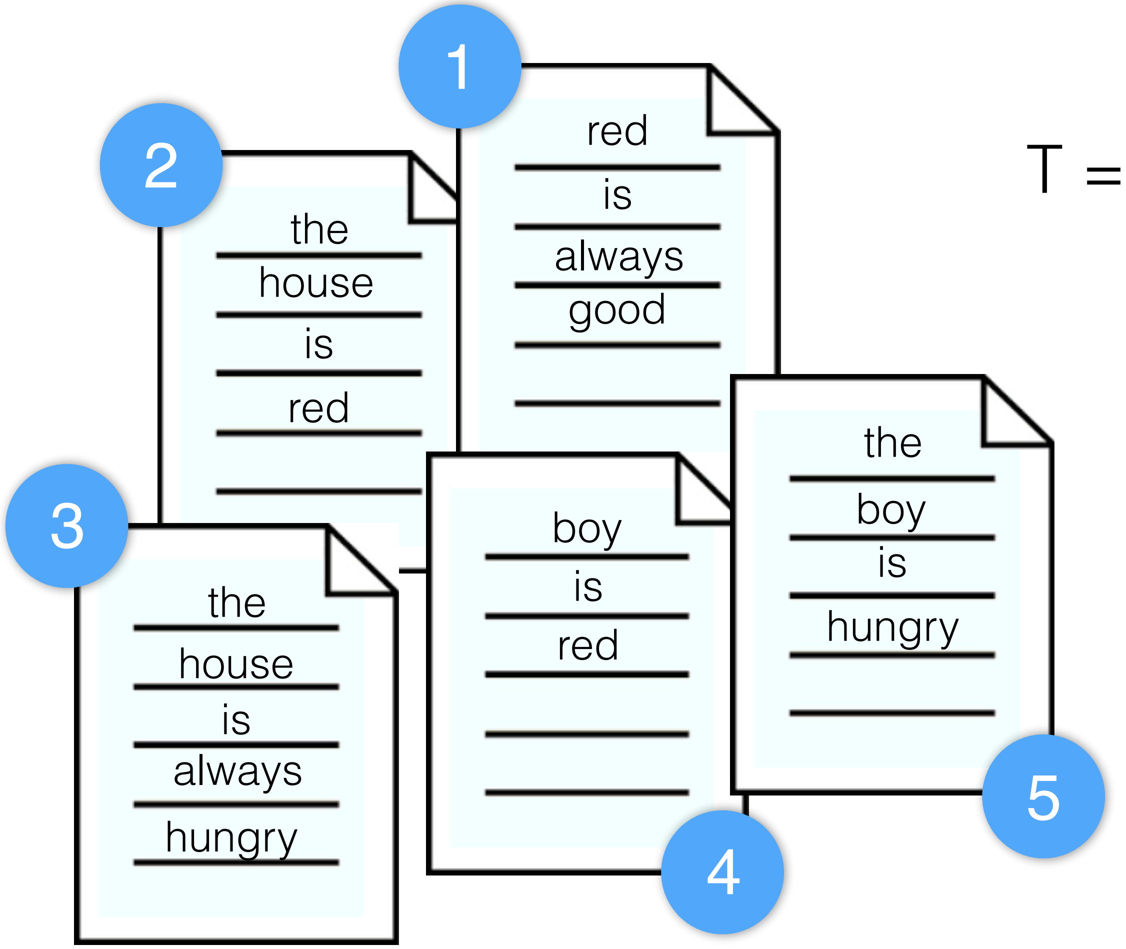

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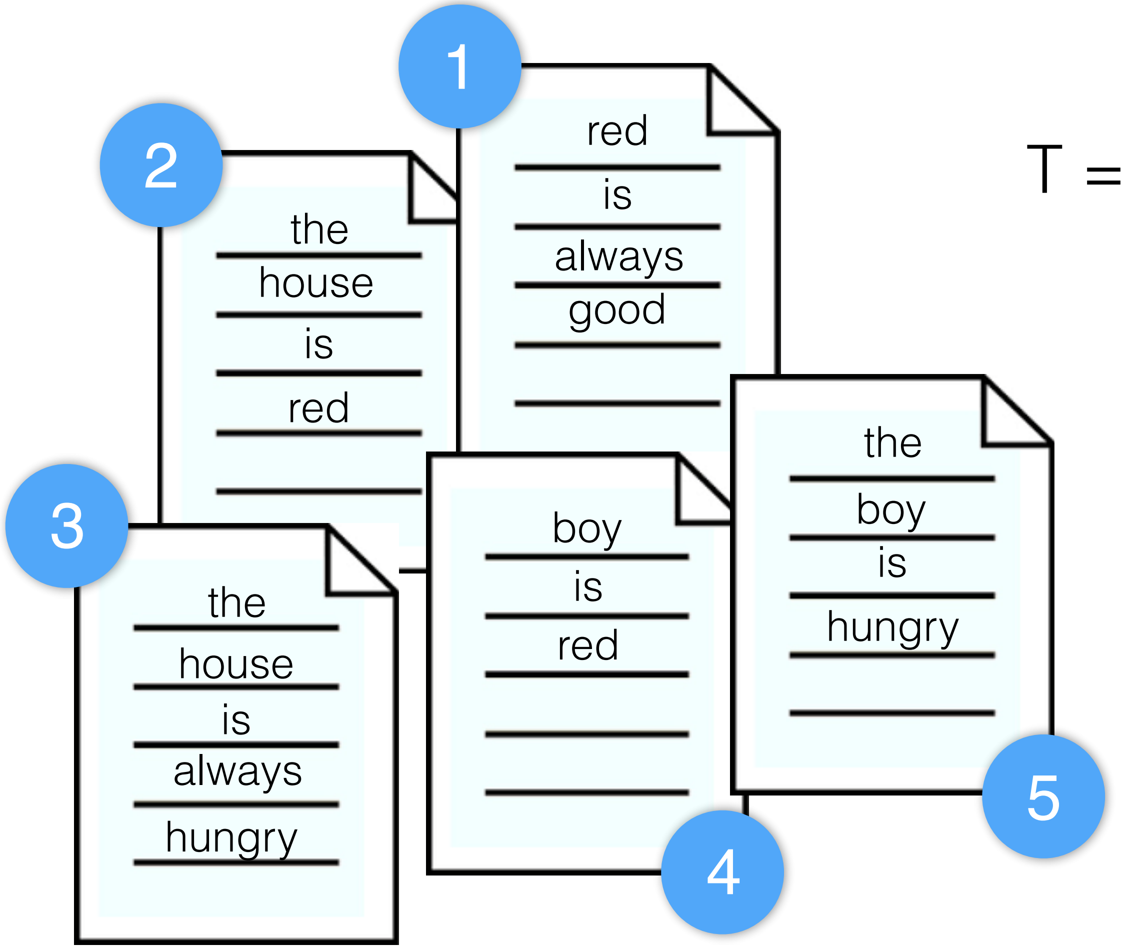

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inverted lists intersection

Genesis - 1970s

Peter Elias
[1923 - 2001]

Robert Fano
[1917 -]

Robert Fano. *On the number of bits required to implement an associative memory*. Memorandum 61, Computer Structures Group, MIT (1971).

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40 years later!

Sebastiano Vigna. *Quasi-succinct indices*.

In Proceedings of the 6-th ACM International Conference on Web Search and Data Mining (WSDM), 83-92 (2013).

Elias-Fano solution

3	1
4	2
7	3
13	4
14	5
15	6
21	7
43	8

Elias-Fano solution

	3	1
	4	2
	7	3
	13	4
	14	5
	15	6
	21	7
u =	43	8

Elias-Fano solution

0 0 0 0 1 1	3	1
0 0 0 1 0 0	4	2
0 0 0 1 1 1	7	3
0 0 1 1 0 1	13	4
0 0 1 1 1 0	14	5
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1 0 1 0 1 1	u = 43	8

Elias-Fano solution

high $\lceil \lg n \rceil$	low $\lceil \lg(u/n) \rceil$		
0 0 0	0 1 1	3	1
0 0 0	1 0 0	4	2
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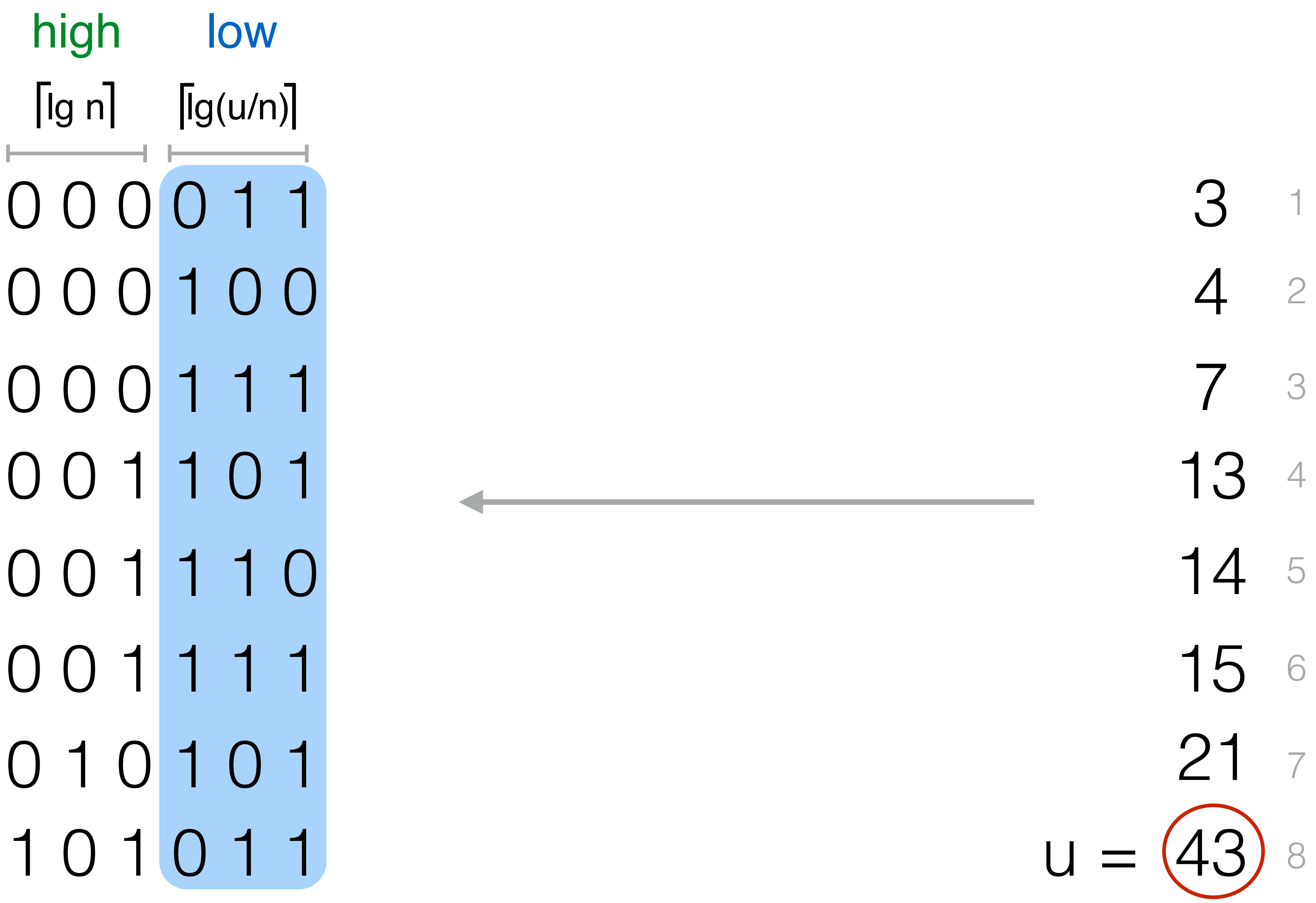

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L = 011100111101110111101011

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	0 0 0	0 1 1	3	1
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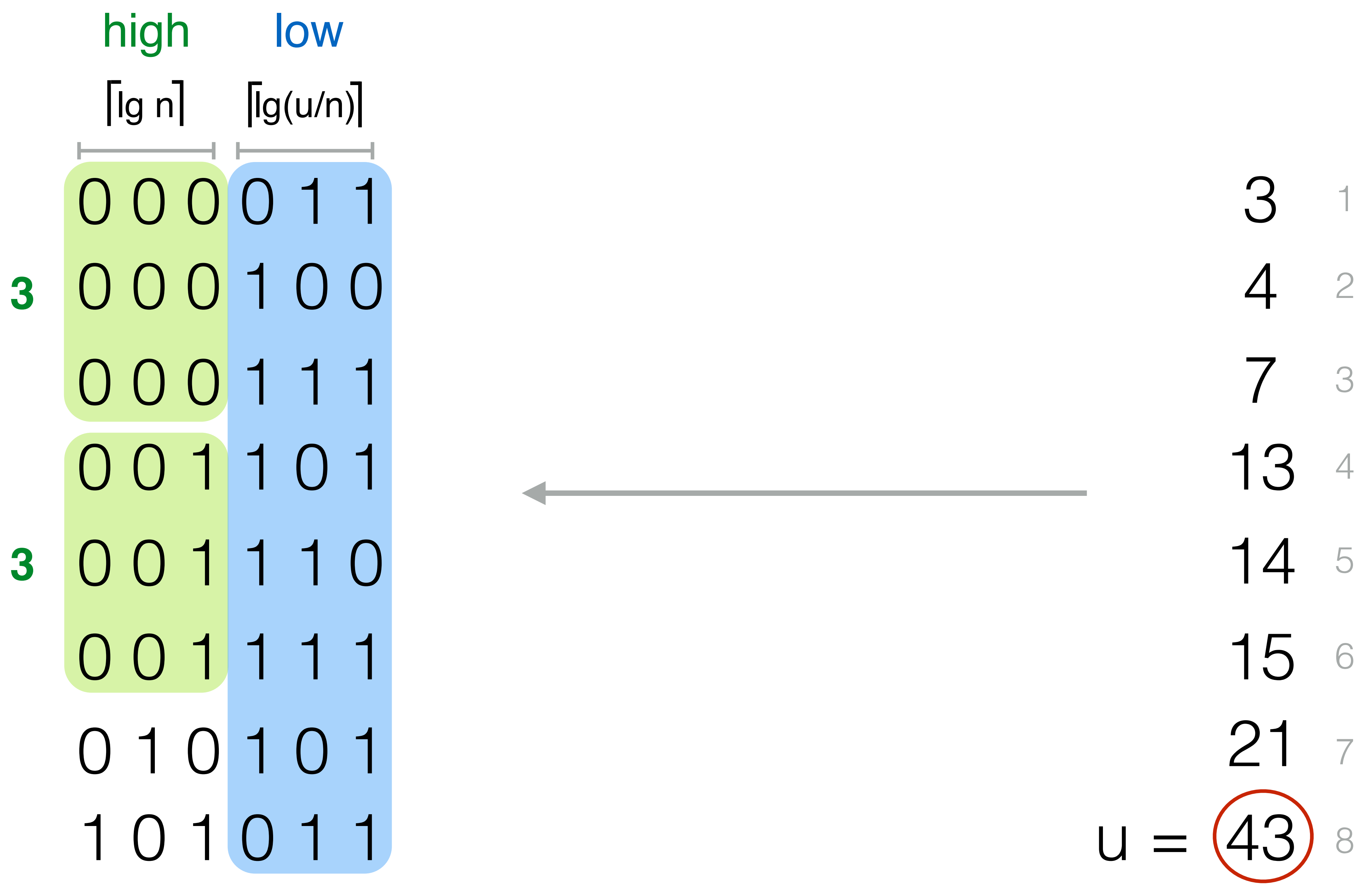

	high	low		
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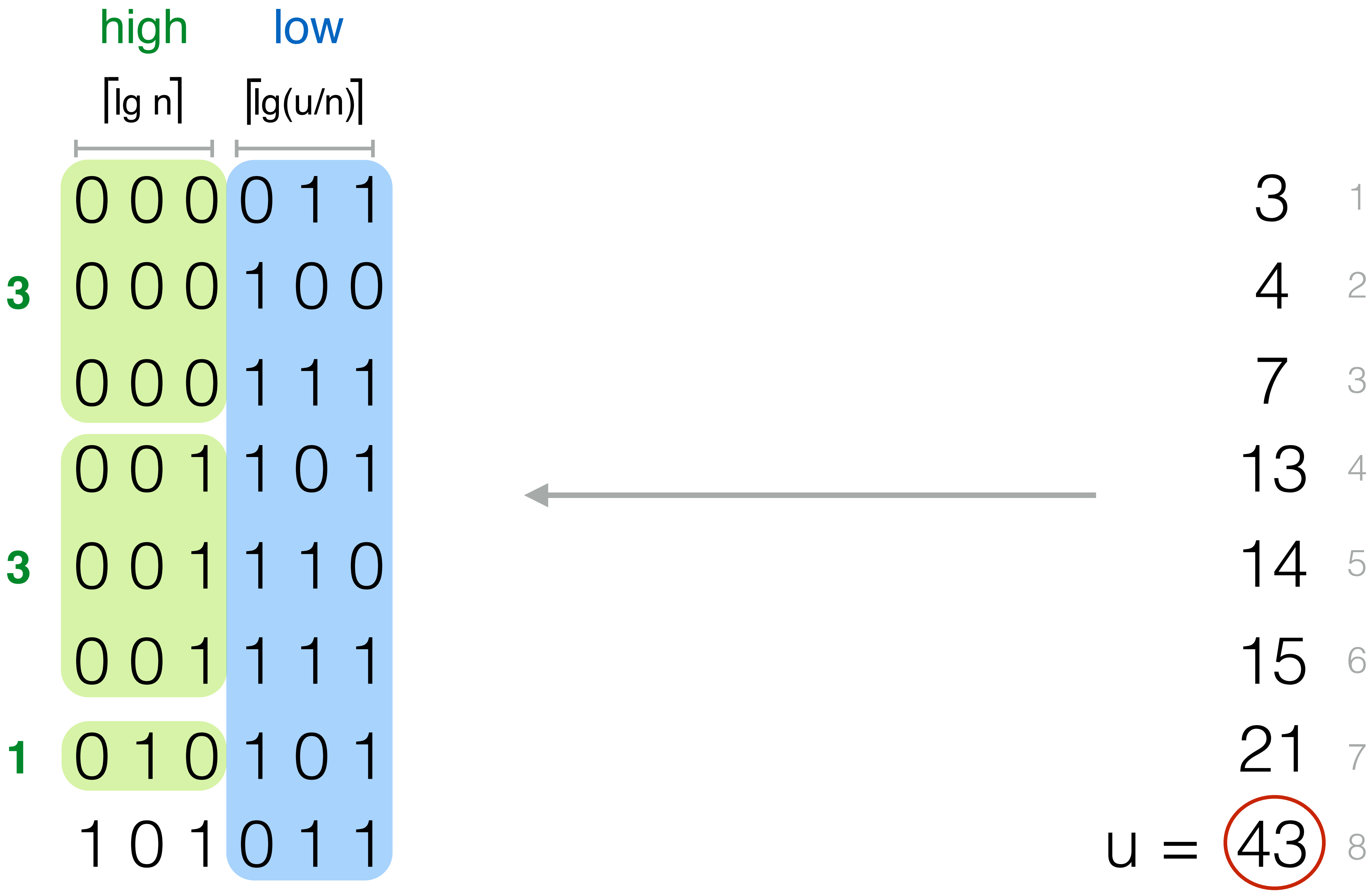

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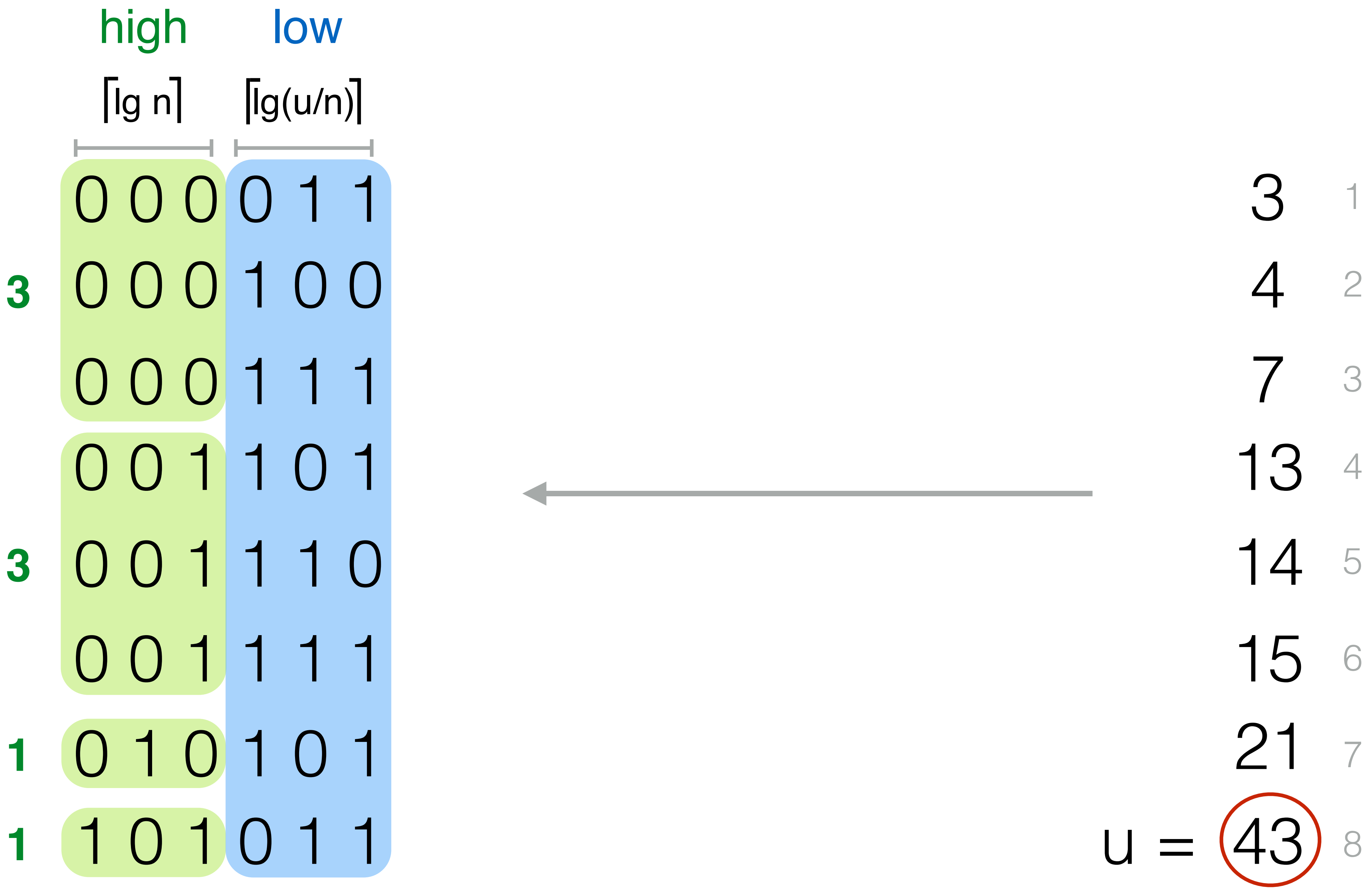

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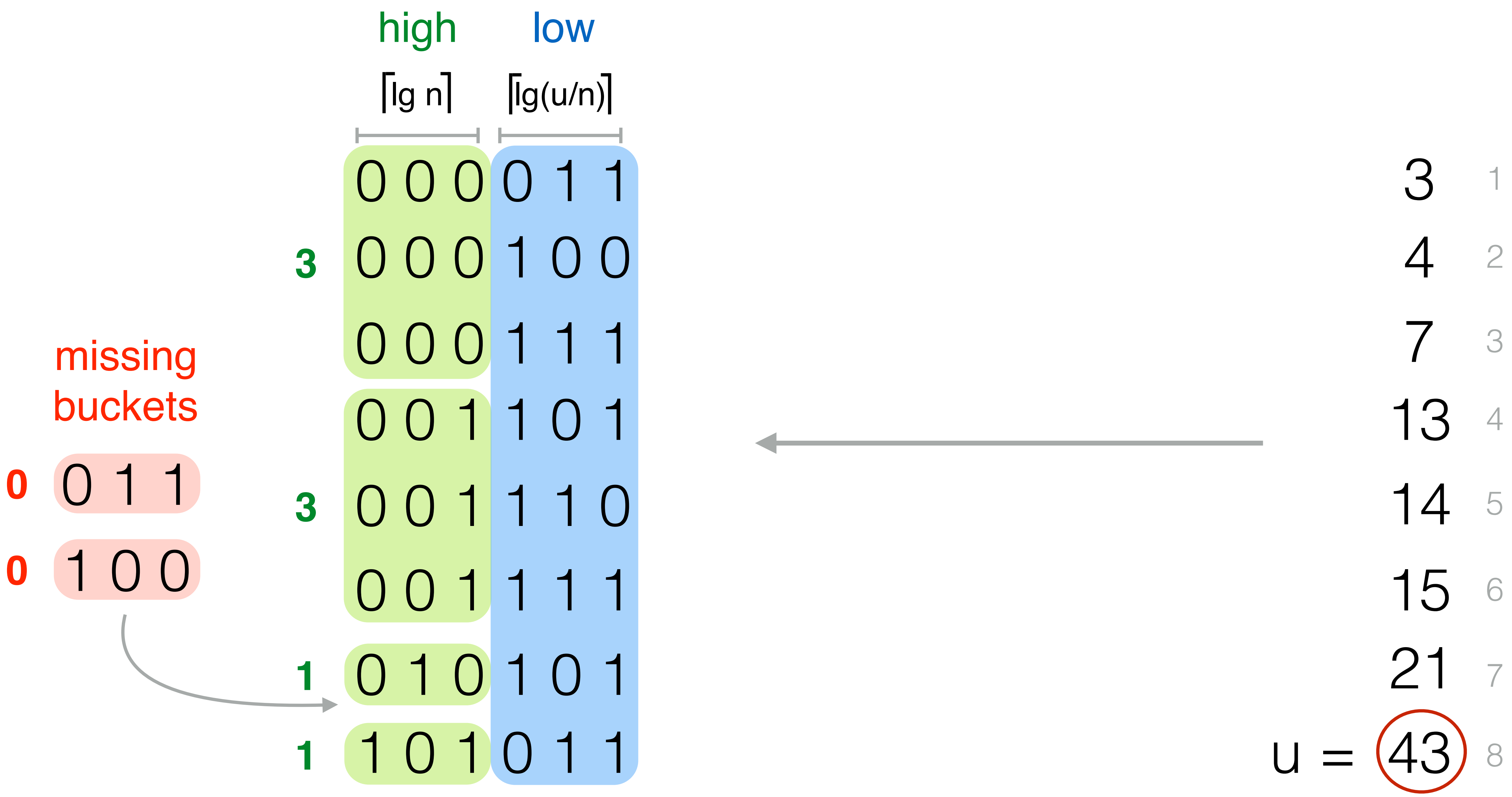

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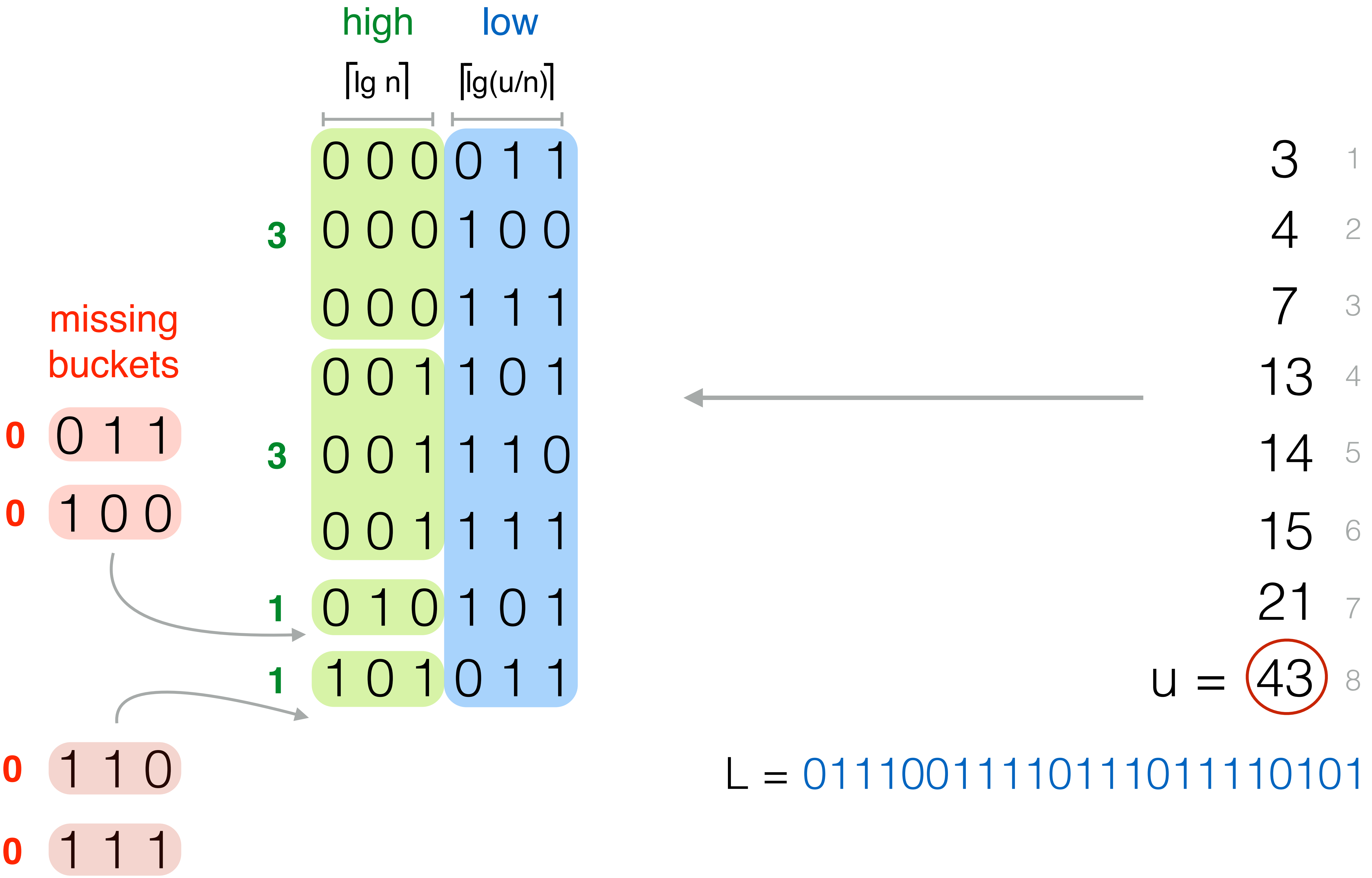

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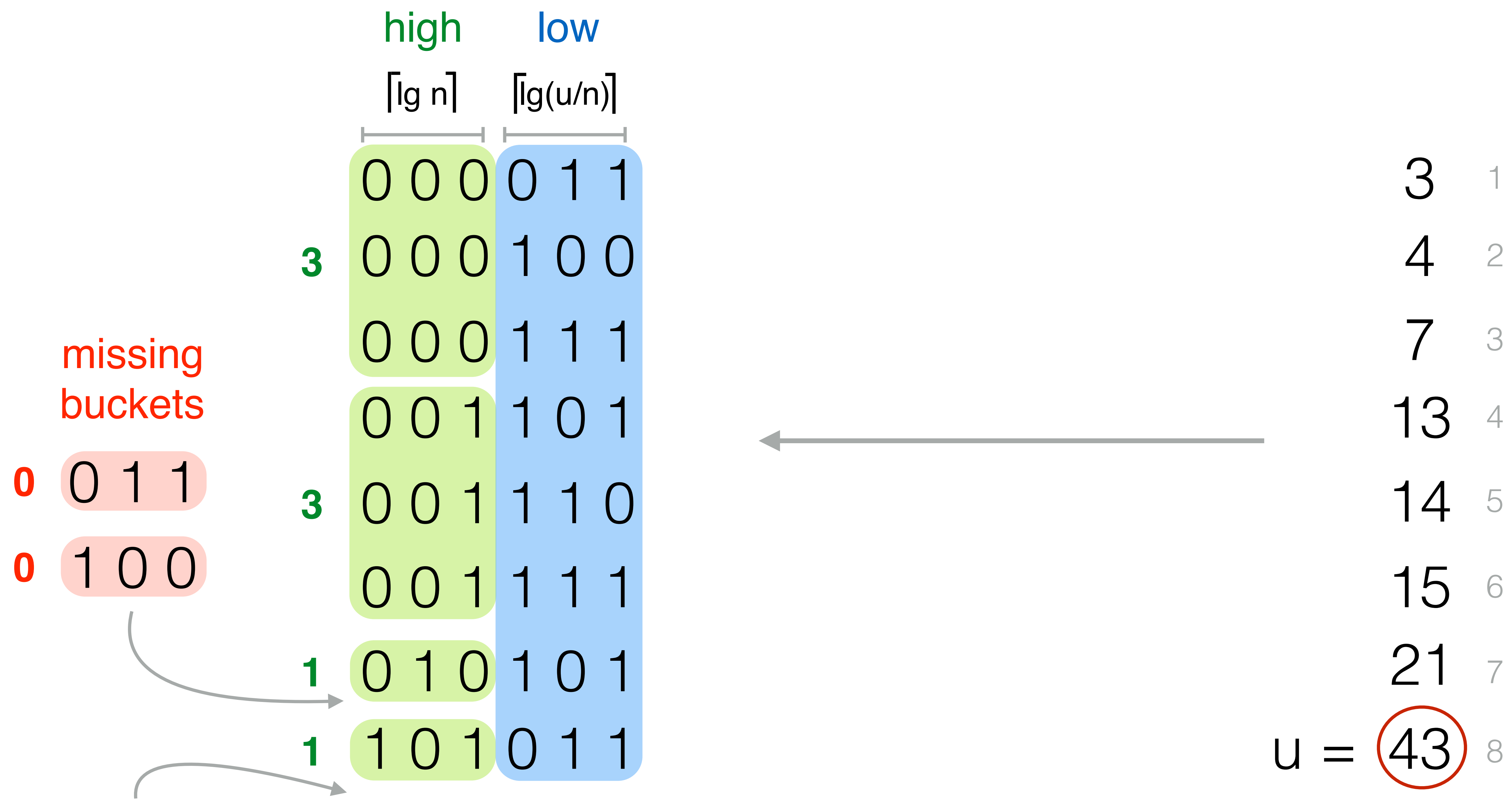

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missing buckets

0 0 1 1

0 1 0 0

0 1 1 0

0 1 1 1

3 3 1 0 0 1 0 0

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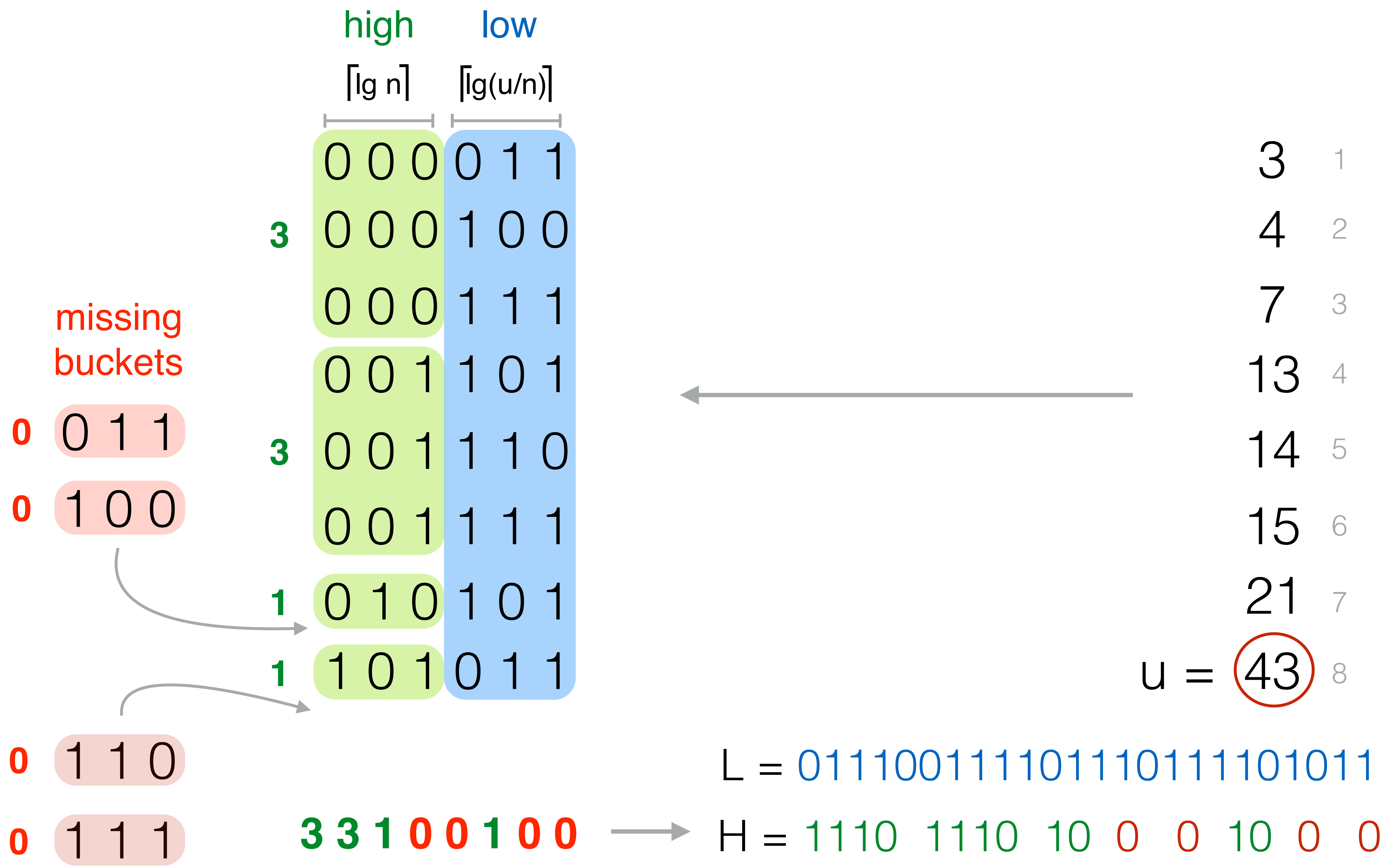

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missing buckets

0 0 1 1

0 1 0 0

0 1 1 0

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3 3 1 0 0 1 0 0

L = 011100111101110111101011

H = 1110 1110 10 0 0 10 0 0

Properties - Space

$EF(S[0,n)) = ?$

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$\lceil \lg(u/n) \rceil$

L = 011100111101110111101011

H = 1110 1110 10 0   0 10 0   0

Properties - Space

1

$$EF(S[0,n)) = n \left\lceil \lg \frac{u}{n} \right\rceil$$

$$L = \overbrace{011}^{\lceil \lg(u/n) \rceil}100111101110111101011$$

$$H = 1110 \quad 1110 \quad 10 \quad 0 \quad 0 \quad 10 \quad 0 \quad 0$$

Properties - Space

$$\mathrm{EF}(S[0,n)) = n\left\lceil \lg \frac{u}{n} \right\rceil$$

$\lceil \lg(u/n) \rceil$

L = 011100111101110111101011

H = 1110 1110 10 0   0 10 0   0

n ones

Properties - Space

$$EF(S[0,n)) = n \left\lceil \lg \frac{u}{n} \right\rceil$$

$$\overbrace{}^{\lceil \lg(u/n) \rceil}$$

L = 011100111101110111101011

H = 1110 1110 10 0   0 10 0   0

n ones

We store a 0 whenever we change bucket.

Properties - Space

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3 6 10 11

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3 6 1011 17

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With possible repetitions!
(*weak* monotonicity)

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000**1**00**1**000**11**00000**1**

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0001001000011000001

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(less than half a bit away [Elias-1974])

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queries in $O\left(\lg \frac{u}{n}\right)$ worst-case

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Succinct rank/select

Definition

Given a bitvector B of n bits:

$\texttt{rank}_{0/1}(\texttt{i})$ = # of 0/1 in [0,$\texttt{i}$)

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B = 101011010101111010110101

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`rank`$_{0}$`(5) = 2`

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$\mathtt{rank}_0(5) = 2$

$\mathtt{rank}_1(7) = 4$

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$\texttt{rank}_{0}(5) = 2$ $\texttt{select}_{0}(5) = 10$

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Examples

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$\texttt{rank}_{0}(5) = 2$ $\texttt{select}_{0}(5) = 10$

$\texttt{rank}_{1}(7) = 4$ $\texttt{select}_{1}(7) = 11$

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Examples

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$\mathrm{rank}_{0}(5) = 2$ $\mathrm{select}_{0}(5) = 10$

$\mathrm{rank}_{1}(7) = 4$ $\mathrm{select}_{1}(7) = 11$

Relations

$$\mathrm{rank}_{1/0}(\mathrm{select}_{0/1}(i)) = \mathrm{select}_{0/1}(i) - i$$

$$\mathrm{rank}_{0/1}(\mathrm{select}_{0/1}(i)) = i\text{-}1$$

$$\mathrm{rank}_{0/1}(i) + \mathrm{rank}_{1/0}(i) = i$$

Succinct rank/select

O(1)-solutions with o(n) bits

rank	(multi)-layered index + precomputed table	[Jacobson-1989]
select	three-level directory tree	[Clark-1996]

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2^{30} bits → ~**67**% more bits!

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Nowadays *practical* solutions are based on [Vigna-2008, Zhou *et al.*-2013]:

- broadword programming
- interleaving
- Intel hardware **popcnt** instruction:

`Long().bitCount(x)` in Java

`__builtin_popcountl(x)` in C/C++

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- broadword programming
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- Intel hardware **`popcnt`** instruction:

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`rank` → ~**3**% more bits

`select` → ~**0.39**% more bits

with practical constant-time `selection`

access example

S = [3, 4, 7, 13, 14, 15, 21, 43]

1 2 3 4 5 6 7 8

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$$\text{access}(i) = \text{rank}_0(\text{select}_1(i)) = \text{select}_1(i) - i$$

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$k = \lceil \lg(u/n) \rceil$

$$\mathtt{access}(i) = \mathtt{select}_1(i) - i \ll k \mid L[(i-1)k, ik)$$

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L = 011100111101110111101011

$k = \lceil \lg(u/n) \rceil$

access(i) = select_1(i) - i << k | L[(i-1)k,ik)

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`access(4)` = S[4] = 001 101

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```
access(i) = select₁(i) - i << k | L[(i-1)k,ik)
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Complexity: O(1)

access(i) = select₁(i) - i << k | L[(i-1)k,ik)

successor **example**

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successor **example**

S = [3, 4, 7, 13, 14, 15, 21, 43]

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successor(12) = ?

H = 1110111010001000

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`successor` example

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001100

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successor(12) = ?

$h_{12} = \boxed{001}100$

$p_1 = \text{select}_0(h_x) - h_x$

$p_2 = \text{select}_0(h_x+1) - h_x - 1$

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H = ~~1110~~1110~~10001000~~

L = ~~011100111~~101110111~~101011~~

p_1 p_2

→

binary search
in [p_1,p_2)

successor **example**

$S = [3, 4, 7, 13, 14, 15, 21, 43]$

1 2 3 4 5 6 7 8

$\texttt{successor}(12) = 13$

$h_{12} = \boxed{001}100$

$p_1 = \texttt{select}_0(h_x) - h_x$

$p_2 = \texttt{select}_0(h_x + 1) - h_x - 1$

H = ~~1110~~1110~~10001000~~

L = ~~011100111~~101110111~~101011~~

p_1 p_2

→ *binary search* in $[p_1, p_2)$

successor **example**

S = [3, 4, 7, 13, 14, 15, 21, 43]

1 2 3 4 5 6 7 8

successor(12) = 13

$h_{12} = \boxed{001}100$

$p_1 = \mathrm{select}_0(h_x) - h_x$

$p_2 = \mathrm{select}_0(h_x+1) - h_x - 1$

H = ~~1110~~1110~~10001000~~

L = ~~011100111~~101110111~~101011~~

p_1 p_2

→

binary search
in $[p_1, p_2)$

Complexity: $O\left(\lg \frac{u}{n}\right)$

Performance

4 Intel i7-4790K cores (8 threads) clocked at 4Ghz, with 32 GB RAM, running Linux 4.2.0, 64 bits

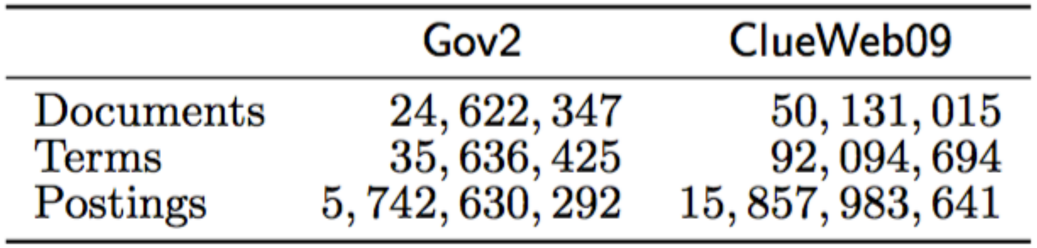
C++11, compiled with `gcc` 5.3.0 with the highest optimisation setting

n	u	`access`	`successor`	iterated `successor`	iterator
~2.4x10^{6}	~1.76x10^{9}	27.6 ns	0.24 μs	7.61 ns	2.34 ns
~10.5x10^{6}	~7.83x10^{9}	41.4 ns	0.29 μs	7.61 ns	2.36 ns

n	uncompressed sequence bytes	Elias-Fano bytes	compression ratio
~2.4x10^{6}	18,787,288	3,530,704	532%
~10.5x10^{6}	83,565,504	15,704,680	532%

Performance

Datasets



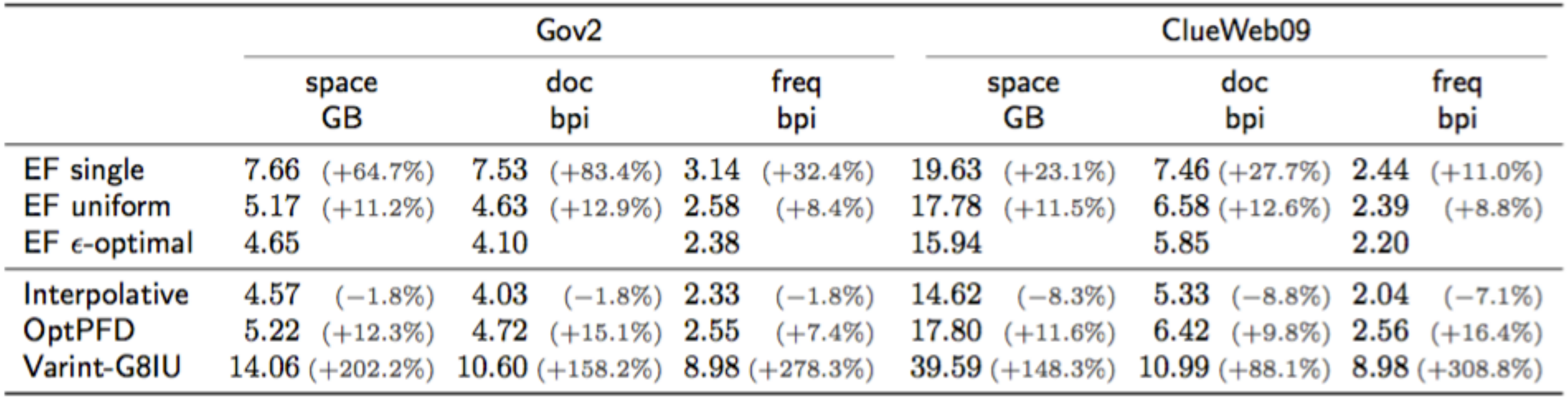
	Gov2	ClueWeb09
Documents	24, 622, 347	50, 131, 015
Terms	35, 636, 425	92, 094, 694
Postings	5, 742, 630, 292	15, 857, 983, 641

24 Intel Xeon E5-2697 Ivy Bridge cores (48 threads) clocked at 2.70Ghz, with 64 GB RAM, running Linux 3.12.7, 64 bits

C++11, compiled with `gcc` 4.9 with the highest optimisation setting

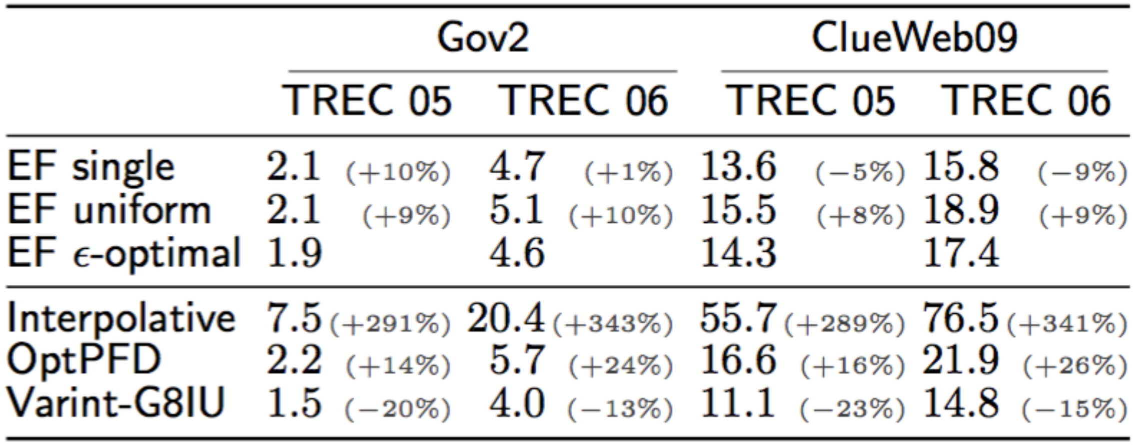
Numbers from [Ottaviano and Venturini-2014].

Space



	Gov2			ClueWeb09		
	space GB	doc bpi	freq bpi	space GB	doc bpi	freq bpi
EF single	7.66 (+64.7%)	7.53 (+83.4%)	3.14 (+32.4%)	19.63 (+23.1%)	7.46 (+27.7%)	2.44 (+11.0%)
EF uniform	5.17 (+11.2%)	4.63 (+12.9%)	2.58 (+8.4%)	17.78 (+11.5%)	6.58 (+12.6%)	2.39 (+8.8%)
EF ϵ-optimal	4.65	4.10	2.38	15.94	5.85	2.20
Interpolative	4.57 (−1.8%)	4.03 (−1.8%)	2.33 (−1.8%)	14.62 (−8.3%)	5.33 (−8.8%)	2.04 (−7.1%)
OptPFD	5.22 (+12.3%)	4.72 (+15.1%)	2.55 (+7.4%)	17.80 (+11.6%)	6.42 (+9.8%)	2.56 (+16.4%)
Varint-G8IU	14.06 (+202.2%)	10.60 (+158.2%)	8.98 (+278.3%)	39.59 (+148.3%)	10.99 (+88.1%)	8.98 (+308.8%)

AND queries (timings are in milliseconds)



	Gov2		ClueWeb09	
	TREC 05	TREC 06	TREC 05	TREC 06
EF single	2.1 (+10%)	4.7 (+1%)	13.6 (−5%)	15.8 (−9%)
EF uniform	2.1 (+9%)	5.1 (+10%)	15.5 (+8%)	18.9 (+9%)
EF ϵ-optimal	1.9	4.6	14.3	17.4
Interpolative	7.5 (+291%)	20.4 (+343%)	55.7 (+289%)	76.5 (+341%)
OptPFD	2.2 (+14%)	5.7 (+24%)	16.6 (+16%)	21.9 (+26%)
Varint-G8IU	1.5 (−20%)	4.0 (−13%)	11.1 (−23%)	14.8 (−15%)

Killer applications

1. Inverted Indexes

Sebastiano Vigna. *Quasi-succinct indices*. In Proceedings of the 6-th ACM International Conference on Web Search and Data Mining (WSDM), 83-92 (2013).

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2. Social Networks

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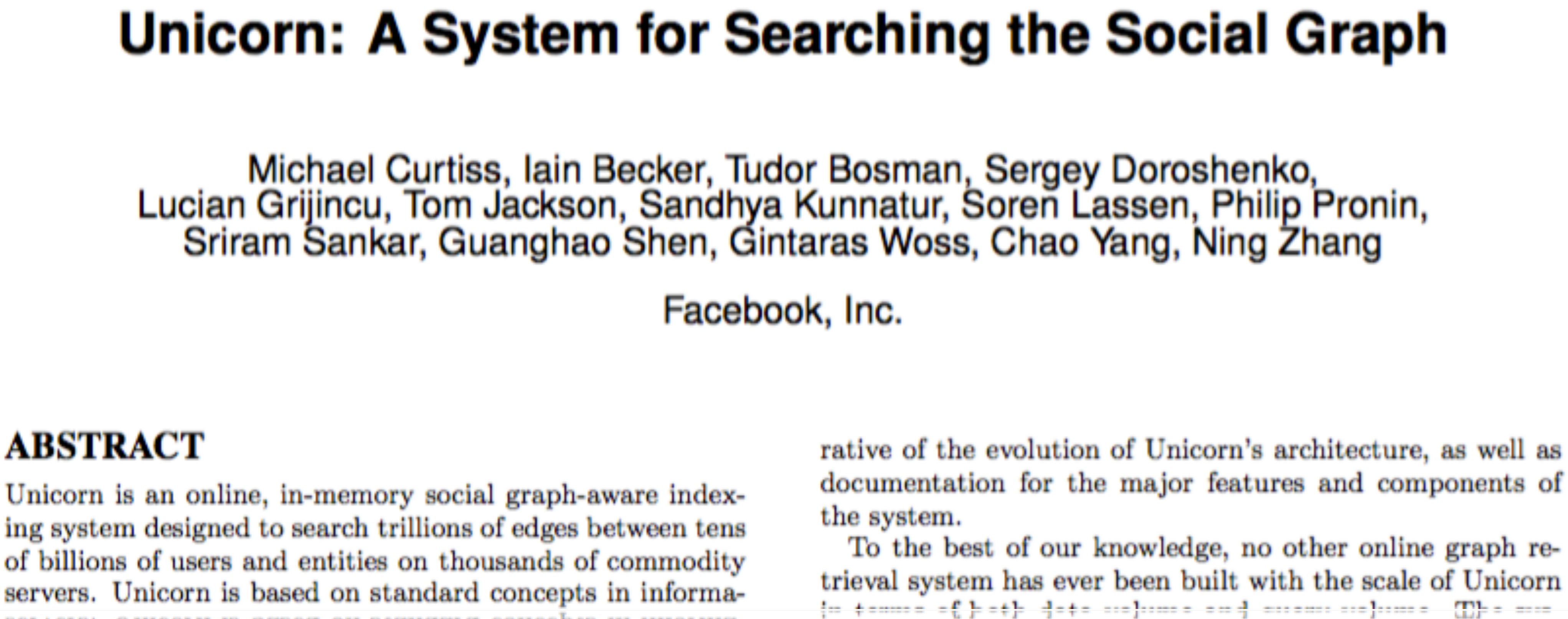
2. Social Networks



Unicorn: A System for Searching the Social Graph

Michael Curtiss, Iain Becker, Tudor Bosman, Sergey Doroshenko,
Lucian Grijincu, Tom Jackson, Sandhya Kunnatur, Soren Lassen, Philip Pronin,
Sriram Sankar, Guanghao Shen, Gintaras Woss, Chao Yang, Ning Zhang

Facebook, Inc.

ABSTRACT

Unicorn is an online, in-memory social graph-aware indexing system designed to search trillions of edges between tens of billions of users and entities on thousands of commodity servers. Unicorn is based on standard concepts in informa-

rative of the evolution of Unicorn's architecture, as well as documentation for the major features and components of the system.

To the best of our knowledge, no other online graph retrieval system has ever been built with the scale of Unicorn

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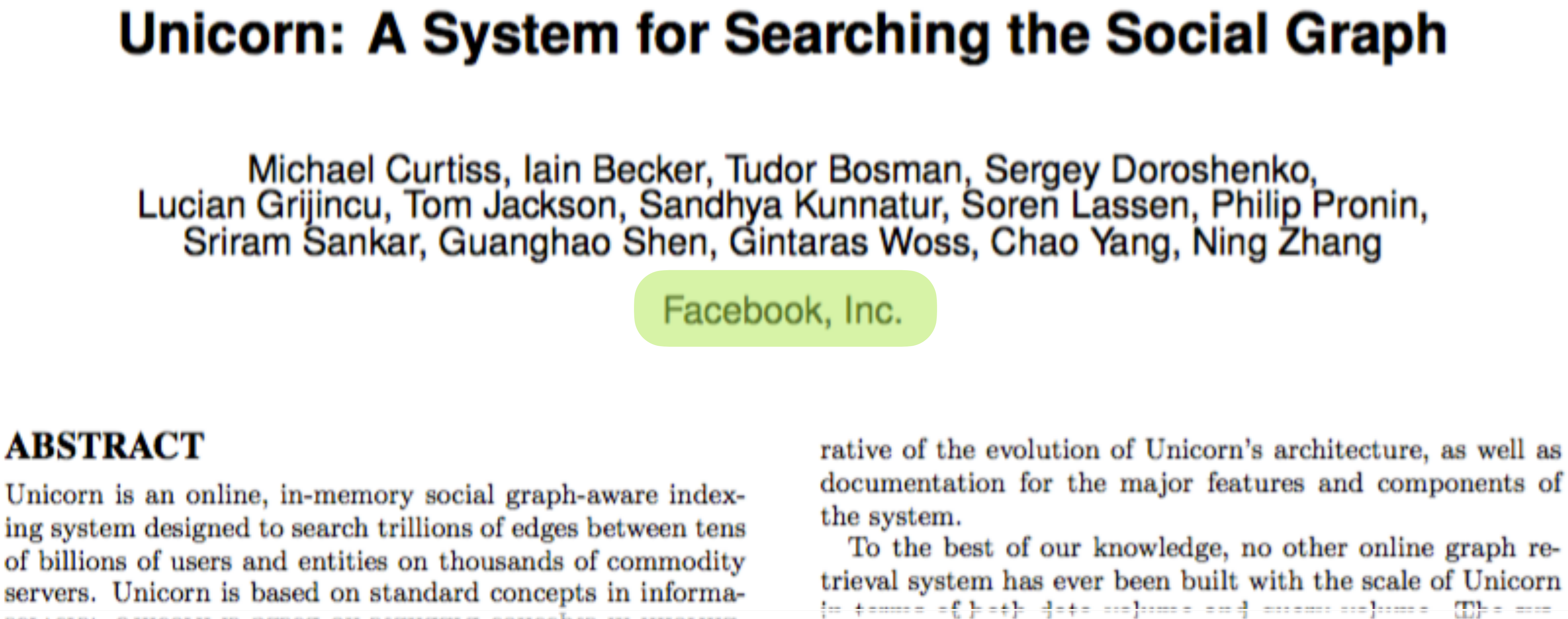

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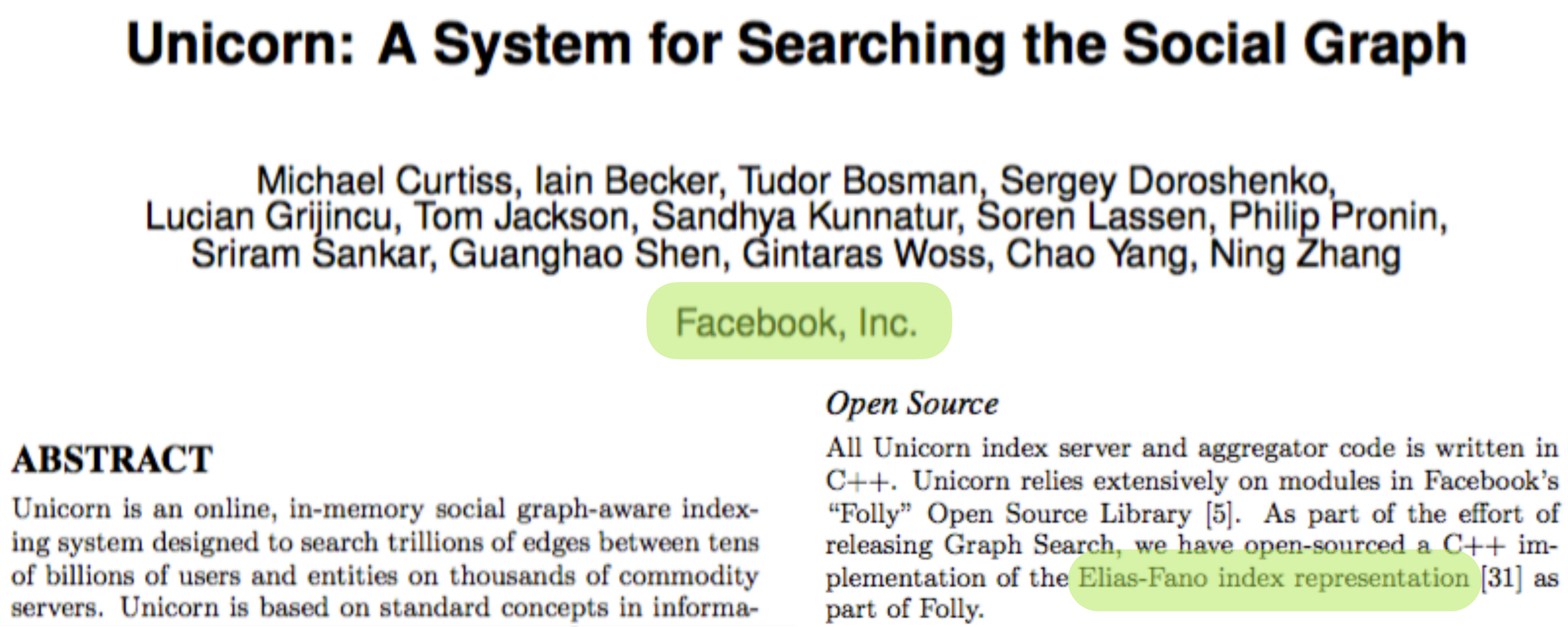

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Open Source

All Unicorn index server and aggregator code is written in C++. Unicorn relies extensively on modules in Facebook's "Folly" Open Source Library [5]. As part of the effort of releasing Graph Search, we have open-sourced a C++ implementation of the Elias-Fano index representation [31] as part of Folly.

Available Implementations

Library	Author(s)	Link	Language
folly	Facebook, Inc.	https://github.com/facebook/folly	C++
sdsl	Simon Gog	https://github.com/simongog/sdsl-lite	C++
ds2i	Giuseppe Ottaviano Rossano Venturini Nicola Tonellotto	https://github.com/ot/ds2i	C++
Sux	Sebastiano Vigna	http://sux.di.unimi.it	Java/C++

Summary

Elias-Fano encodes *monotone integer sequences* in *space close to the information theoretic minimum*, while allowing *powerful search operations*, namely `predecessor/successor` queries and random `access`.

Successfully applied to crucial problems, such as *inverted indexes* and *social graphs* representation.

Several *optimized* software implementations are available.

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Thanks for your attention, time, patience!

Any questions?