## Sparse and Skew Hashing of K-Mers*

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## The K-Mer Dictionary Problem

- K-Mer. $\mathrm{A} k$-mer is a string of length $k$ over the DNA alphabet $\{\mathrm{A}, \mathrm{C}, \mathrm{G}, \mathrm{T}\}$.
- We are given a large DNA string (e.g., a genome or a pan-genome) and let $K$ be the set of all its $n$ distinct $k$-mers.

Example: The human genome (GRCh38) has $>2.5$ B distinct $k$-mers for $k=31$.

- Problem. We want to build a dictionary for $K$ so that the following operations are efficient:
$-i=\operatorname{Lookup}(g)$, where $0 \leq i<n$ if the $k$-mer $g \in K$ or $i=-1$ otherwise;
- return the $k$-mer $g=\operatorname{Access}(i)$ if $0 \leq i<n$.
(Other operations of interest are iteration and streaming membership queries. See the paper for details.)


## K-Mer Applications

- Software tools based on $k$-mers are predominant in Bioinformatics.
- Many applications:
- genome assembly
- variant calling
- pan-genome analysis
- meta-genomics
- sequence comparison/alignment
- ...


## de Bruijn Graphs

- Fact. Equivalence between a set of $k$-mers and a de Bruijn graph (dBG).
- There are efficient software tools to run the following pre-processing flow.

- BCALM [Chikhi et al., 2016]
- Cuttlefish [Khan and Patro, 2021]
- A collection of DNA strings with no duplicate $k$-mers.
- Efficient heuristic method to reduce the number of bases, e.g, UST [Rahman and Medvedev, 2020].


## Super-k-Mers

- Property. Consecutive $k$-mers are likely to have the same minimizer [Roberts et al., 2004] the smallest sub-string of length $m \leq k$ according to a given order $R$.

Example for $k=13$ and $m=4$ :
ACGGTAGAACCGATTCAAATTCGATCGATTAATTAGAGCGATAAC... ACGGTAGAACCGA
CGGTAGAACCGAT GGTAGAACCGATT GTAGAACCGATTC TAGAACCGATTCA AGAACCGATTCAA GAACCGATTCAAA AACCGATTCAAAT

- Super-k-mer. [Li et al., 2013] Given a string, a super-k-mer is a maximal sequence of consecutive $k$-mers having the same minimizer.


## Super-k-Mers

- Observation 1. Since consecutive $k$-mers are likely to have the same minimizers, there are far fewer super- $k$-mers than $k$-mers - approx. $(k-m+2) / 2$ times less for random minimizers $\rightarrow$ sparse indexing.
- Observation 2. A super- $k$-mer of length $s$ is a space-efficient representation of the set of its constituent $s-k+1 k$-mers: $2 s /(s-k+1)$ vs. $2 k$ bits/ $k$-mer. If $s$ is sufficiently large and/or we have long chains of super- $k$-mers, the cost becomes approx. 2 bits/k-mer.

This super- $k$-mer costs $2 \times 19=38$ bits for $7 k$-mers
( $5.43 \mathrm{bits} / k$-mer vs. $2 \times 13=26 \mathrm{bits} / k$-mer).
Example for $k=13$ and $m=4$ :
ACGGTAGAACCGATTCAAATTCGATCGATTAATT...
$\uparrow$
ACGGTAGAACCGATTCAAA $s=19$

AACCGATTCAAATTCGATCGATTA $s=24$

This chain is of length 31 and costs $2 \times 31=62$ bits for $19 k$-mers ( 3.26 bits/k-mer).

## Sparse Hashing

- Q. How to index super- $k$-mers?
- Do not break the chains of super- $k$-mers to avoid wasting $2(k-1)$ bits per super- $k$-mer.
- Locate super- $k$-mers with an array of offsets into the strings, indexed by a minimal perfect hash function (MPHF) on the minimizers.
- Upon Lookup $(g)$ : if $r$ is the minimizer of $g$, locate and scan the "bucket" of $r$ - the set of super- $k$-mers that have minimizer $r$.


AGATGATGAACCTGAAAACATCCTGAAAATCGTCAAAGAATGGCGGCGTTCACAGGGGCTACCCTTGTTTAAAGACTCTAAATAAAGTA.ATTTTCAGGATG TTTTCAGGTTCATCATCTCCCTTCTTTGCAGGATAGTAGATAAGATCGCTCATCAACGGATGTTGTGTAATTCTGGTAAGATGTTCTTCTAGATCATCCCAA TATTTGTCAAGCACTTCCCCTTTTAATTGAGCGTTATCCCCGG.AGATGATGAACCTGAAAACATCCTGAAAATTGTCAAAGAATGGCGGCGTTCACAGGGG CTA.ATTGTCAAAGAATGGCGGCGTTCACAGGGGTTACCCTTGTTTAAAGACTCTAAATAAAGTAGATAATAAAACTATATATGGAACATCATCGCATCTGG

## Skew Hashing

- Problem. Some buckets can be very large.

For example on the human genome (GRCh38), for $k=31$ and $m=20$ : largest bucket size can be as large as $3.6 \times 10^{4}$.

- Property. Minimizers have a (very) skew distribution for sufficiently long length.

Bucket size distribution (\%) for $k=31$ and the first $n=10^{9} k$-mers of the human genome, by varying minimizer length $m$.

| size $/ m$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 13.7 | 19.8 | 29.7 | 42.4 | 61.5 | 79.5 | 89.8 | 94.4 | 96.3 | 97.1 | 97.5 |
| 2 | 7.5 | 10.6 | 14.4 | 17.7 | 19.4 | 13.6 | 7.3 | 3.9 | 2.4 | 1.7 | 1.4 |
| 3 | 5.2 | 7.3 | 8.8 | 10.4 | 8.4 | 3.7 | 1.4 | 0.8 | 0.5 | 0.4 | 0.4 |
| 4 | 4.0 | 5.5 | 6.0 | 7.0 | 4.1 | 1.3 | 0.5 | 0.3 | 0.2 | 0.2 | 0.2 |
| 5 | 3.2 | 4.4 | 4.5 | 5.0 | 2.2 | 0.6 | 0.3 | 0.2 | 0.1 | 0.1 | 0.1 |

On the full human genome (GRCh38),
for $k=31$ and $m=20$ :
2,505,445,761k-mers
421,845, 806 minimizers
$388,018,280(91.98 \%)$ only appear once!

## Skew Hashing

- We fix an integer $\ell$ : by virtue of the skew distribution, the fraction of buckets having more than $2^{\ell}$ super- $k$-mers is small.
- So, we can afford a MPHF over the set of $k$-mers that belong to such super- $k$-mers. The output of the MPHF for a $k$-mer $g$ is the identifier of the super- $k$-mer where $g$ is present. (This identifier can be written in few bits; see the paper for details.)
- Upon Lookup, we will scan one super- $k$-mer only.

Bucket size distribution (\%) for $k=31$ and the first $n=10^{9} k$-mers of the human genome, by varying minimizer length $m$.

| size $/ m$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 13.7 | 19.8 | 29.7 | 42.4 | 61.5 | 79.5 | 89.8 | 94.4 | 96.3 | 97.1 | 97.5 |
| 2 | 7.5 | 10.6 | 14.4 | 17.7 | 19.4 | 13.6 | 7.3 | 3.9 | 2.4 | 1.7 | 1.4 |
| 3 | 5.2 | 7.3 | 8.8 | 10.4 | 8.4 | 3.7 | 1.4 | 0.8 | 0.5 | 0.4 | 0.4 |
| 4 | 4.0 | 5.5 | 6.0 | 7.0 | 4.1 | 1.3 | 0.5 | 0.3 | 0.2 | 0.2 | 0.2 |
| 5 | 3.2 | 4.4 | 4.5 | 5.0 | 2.2 | 0.6 | 0.3 | 0.2 | 0.1 | 0.1 | 0.1 |

$$
\begin{aligned}
& \text { For } \ell=2 \text {, just } \\
& 100.0-(97.1+1.7+0.4+0.2) \%=0.6 \% \text { of } \\
& \text { buckets with more than } 2^{\ell=2}=4 \text { super- } k \text {-mers. }
\end{aligned}
$$

## Benchmarking SSHash

- Code in C++17, compiled with flags: -03 -march=native.
- All experiments are single-threaded.
- We use $\ell=6$ for all experiments.
- We use $m=17,17,20,20$ for respectively Cod, Kestrel, Human, and Bacterial. (A good rule of thumb is $m=\left\lceil\log _{4}(N)\right\rceil+1$ or $m=\left\lceil\log _{4}(N)\right\rceil+2$.)

Some basic statistics for the datasets used in the experiments, for $k=31$, such as number of: $k$-mers ( $n$ ), paths ( $p$ ), and bases $(N)$.

| Dataset | $n$ | $p$ | $N$ | $\left\lceil\log _{2}(N)\right\rceil$ |
| :--- | ---: | ---: | ---: | :---: |
| Cod | $502,465,200$ | $2,406,681$ | $574,665,630$ | 30 |
| Kestrel | $1,150,399,205$ | 682,344 | $1,170,869,525$ | 31 |
| Human | $2,505,445,761$ | $13,014,641$ | $2,895,884,991$ | 32 |
| Bacterial | $5,350,807,438$ | $26,449,008$ | $6,144,277,678$ | 33 |

NOTE: We used BCALM (v2)
[Chikhi et al., 2016] to build the compacted dBG and then UST [Rahman and Medvedev, 2020] to compute the stitched unitigs.

## Competitors

- dBG-FM [Chikhi et al., 2014]: FM-index [Ferragina and Manzini, 2000]
- Pufferfish [Almodaresi et al., 2018]: MPHF
- Blight [Marchet et al., 2021]: MPHF+minimizers


## Overall Comparison - Space and Lookup

Dictionary space in total GB and average bits/k-mer (bpk).

| Dictionary | Cod |  | Kestrel |  | Human |  | Bacterial |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | GB | bpk | GB | bpk | GB | bpk | GB | bpk |
| dBG-FM, $s=128$ | 0.22 | 3.48 | 0.44 | 3.07 | - | - | - | - |
| dBG-FM, $s=64$ | 0.27 | 4.38 | 0.55 | 3.86 | - | - | - | - |
| dBG-FM, $s=32$ | 0.39 | 6.16 | 0.78 | 5.43 | - | - | - | - |
| Pufferfish, sparse | 1.75 | 27.80 | 3.69 | 25.66 | 8.87 | 28.32 | 18.91 | 28.28 |
|  | 1.49 | 23.70 | 3.37 | 23.40 | 7.50 | 23.96 | 16.09 | 24.06 |
| Pufferfish, dense | 2.69 | 42.76 | 5.97 | 41.54 | 14.11 | 45.04 | 30.70 | 45.89 |
|  | 2.43 | 38.66 | 5.65 | 39.28 | 12.74 | 40.68 | 27.88 | 41.68 |
| Blight, $b=4$ | 0.91 | 14.53 | 2.16 | 15.00 | 5.04 | 16.11 | 11.40 | 17.04 |
| Blight, $b=2$ | 1.04 | 16.57 | 2.45 | 17.04 | 5.67 | 18.13 | 12.74 | 19.05 |
| Blight, $b=0$ | 1.17 | 18.61 | 2.74 | 19.06 | 6.32 | 20.17 | 14.12 | 21.11 |
| SSHash, regular | 0.44 | 6.98 | 0.93 | 6.48 | 2.59 | 8.28 | 5.50 | 8.22 |
| SSHash, canonical | 0.50 | 7.92 | 1.00 | 7.30 | 2.94 | 9.39 | 6.17 | 9.22 |

Dictionary Lookup time in average $\mathrm{ns} / k$-mer.

| Dictionary | Cod |  | Kestrel |  | Human |  | Bacterial |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Lkp ${ }^{+}$ | $\mathrm{Lkp}^{-}$ | Lkp ${ }^{+}$ | $\mathrm{Lkp}^{-}$ | Lkp ${ }^{+}$ | $\mathrm{Lkp}^{-}$ | Lkp ${ }^{+}$ | $\mathrm{Lkp}^{-}$ |
| dBG-FM, $s=128$ | 22,980 | 16,501 | 23,934 | 16,764 | - | - | - | - |
| dBG-FM, $s=64$ | 15,013 | 10,919 | 15,929 | 11,462 | - | - |  | - |
| dBG-FM, $s=32$ | 11,386 | 7929 | 11,703 | 8073 | - | - | - | - |
| Pufferfish, sparse | 1110 | 700 | 5456 | 769 | 13,656 | 862 | 27,748 | 983 |
| Pufferfish, dense | 624 | 439 | 635 | 485 | 720 | 519 | 816 | 582 |
| Blight, $b=4$ | 2520 | 2751 | 2743 | 3104 | 2820 | 3329 | 3105 | 3913 |
| Blight, $b=2$ | 1800 | 1643 | 1916 | 1820 | 2008 | 1975 | 2095 | 2146 |
| Blight, $b=0$ | 1571 | 1317 | 1692 | 1472 | 1780 | 1610 | 1859 | 1751 |
| SSHash, regular | 1045 | 1158 | 1042 | 1265 | 1338 | 1530 | 1389 | 1780 |
| SSHash, canonical | 834 | 690 | 882 | 781 | 990 | 854 | 1051 | 995 |

- Compared to BWT-based indexes: one order of magnitude faster for "just" $2 x$ more space.
- Compared to other hashing schemes: 2-5x smaller with comparable of faster query time.


## Overall Comparison - Streaming Queries

Query time for streaming membership queries for various dictionaries. The query time is reported as total time in minutes (tot), and average $\mathrm{ns} / k-\mathrm{mer}$ (avg). We also indicate the query file (SRR number) and the percentage of hits. Both high-hit ( $>70 \%$ hits) and low-hit ( $<1 \%$ hits) workloads are considered.

| Dictionary | Cod |  | Kestrel |  | Human |  | Bacterial |  | Cod |  |  | Kestrel |  | Human |  | Bacterial |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | SRR12858649 |  | SRR11449743 |  | SRR5833294 |  | SRR5901135 |  | Dictionary | SRR11449743 |  | SRR12858649 |  | SRR5901135 |  | SRR5833294 |  |
|  | 81.37\% hits |  | 74.60\% hits |  | 91.65\% hits |  | 87.79\% hits |  |  | 0.65 | \% hits |  | \% hits | 0.002 | \% hits | $0.086 \%$ | \% hits |
|  | tot | avg | tot | avg | tot | avg | tot | avg |  | tot | avg | tot | avg | tot | avg | tot | avg |
| Pufferfish, sparse | 0.6 | 214 | 14.1 | 609 | 17.0 | 651 | 9.1 | 691 | Pufferfish, sparse | 14.6 | 627 | 0.9 | 312 | 11.3 | 855 | 25.5 | 975 |
| Pufferfish, dense | 0.2 | 92 | 8.5 | 368 | 10.5 | 402 | 5.3 | 404 | Pufferfish, dense | 8.7 | 374 | 0.2 | 92 | 5.8 | 435 | 13.6 | 518 |
| Blight, $b=4$ | 2.1 | 766 | 32.5 | 1400 | 27.3 | 1041 | 11.4 | 864 | Blight, $b=4$ | 72.2 | 3112 | 6.6 | 2407 | 35.7 | 2704 | 253.2 | 9675 |
| Blight, $b=2$ | 1.2 | 453 | 16.6 | 714 | 17.5 | 670 | 8.6 | 648 | Blight, $b=2$ | 45.9 | 1978 | 3.0 | 1115 | 19.1 | 1445 | 117.7 | 4498 |
| Blight, $b=0$ | 0.8 | 282 | 10.8 | 464 | 11.5 | 440 | 5.8 | 434 | Blight, $b=0$ | 18.1 | 780 | 1.8 | 655 | 14.4 | 1088 | 32.2 | 1232 |
| SSHash, regular | 0.5 | 166 | 6.2 | 267 | 8.2 | 311 | 3.0 | 223 | SSHash, regular | 10.7 | 463 | 0.9 | 314 | 6.2 | 463 | 14.3 | 544 |
| SSHash, canonical | 0.3 | 111 | 5.1 | 219 | 6.7 | 253 | 2.4 | 184 | SSHash, canonical | 5.1 | 220 | 0.4 | 155 | 2.5 | 183 | 6.4 | 244 |

(a) high-hit workload
(b) low-hit workload

## Conclusions

- SSHash is an efficient solution to the K-Mer Dictionary problem: good trade-off between space and time.
- Compared to BWT-based indexes: one order of magnitude faster for "just" 2X more space. Compared to other hashing schemes: $2-5 \mathrm{X}$ smaller with comparable of faster query time.
- Tool-box: spectrum-preserving string sets (SPSSs), minimizers, minimal perfect hashing (MPHF, https://github.com/jermp/pthash), Elias-Fano.
- Ingredients:
- Sparse indexing to obtain good space effectiveness;
- Skew hashing to guarantee fast lookup for "heavy" buckets.
- Code in $\mathrm{C}++17$ is available at: https://github.com/jermp/sshash.

Thank you for the attention!

## Preliminary Observations

- The algorithmic literature about (compressed) string dictionaries is rich of solutions [MartínezPrieto et al., 2016] (e.g., Front-Coding, path-decomposed tries, double-array tries), but are relevant for "generic strings":
- variable-length,
- larger alphabets (e.g., ASCII),
- (usually) no particular properties of the strings to aid compression.
- Since $k$-mers are extracted consecutively from DNA, a $k$-mer following another one shares $k-1$ bases (very low entropy).
ACGGTAGAACCGATTCAAATTCGACGTAGC...
ACGGTAGAACCGA
$\begin{array}{ll}\text { CGGTAGAACCGAT } & \longleftarrow \quad \text { Example for } k=13 \text {. } \\ \text { GGTAGAACCGATT }\end{array}$
GTAGAACCGATTC
TAGAACCGATTCA


## A World of K-Mer Indexes

- Huge research effort produced many types of indexes based on $k$-mers, with different:
- representations (hashing, BWT-based, exact vs. approximate),
- features (e.g., static vs. dynamic),
- space/time trade-offs,
- operations, ecc.
- Recent surveys on this topic:
- Data Structures based on $k$-mers for Querying Large Collections of Sequencing Data Sets Marchet et al., Genome Research, 2020.
- Data Structures to Represent a Set of $k$-long DNA Sequences Chikhi et al., ACM Computing Surveys, 2021.


## de Bruijn Graphs

Fact. Equivalence between a set of $k$-mers and a de Bruijn graph (dBG).

(c) set of stitched (maximal) unitigs

TCATTGGTAACCG
TGCGAA

## Minimizers

- Minimizer. [Roberts et al., 2004] Given a $k$-mer and an order relation $R$, the minimizer of length $m \leq k$ is the smallest $m$-mer of the $k$-mer according to $R$.
- Example. Given $g=$ ACGGTAGAACCGA $(k=13)$ and $m=4$ :

```
ACGG
    CGGT
        GGTA
            GTAG
            TAGA
                AGAA
                GAAC
                    AACC
                    ACCG
                    CCGA
```

```
h(ACGG) = 9842978325
```

h(ACGG) = 9842978325
h(CGGT) = 817612312
h(CGGT) = 817612312
h(GGTA ) = 8265731 \longleftarrow smallest hash code
h(GGTA ) = 8265731 \longleftarrow smallest hash code
h(GTAG) = 478491248
h(GTAG) = 478491248
h(TAGA) = 17491411
h(TAGA) = 17491411
h(AGAA ) = 17148914
h(AGAA ) = 17148914
h(GAAC) = 91815379
h(GAAC) = 91815379
h(AACC) = 645793914
h(AACC) = 645793914
h(ACCG) = 918417644
h(ACCG) = 918417644
h(CCGA) = 814188124
h(CCGA) = 814188124
If $R$ is the lexicographic order.
If $R$ is defined by a random hash function $h$.

```

\section*{Minimal Perfect Hashing}

MPHF. Given a set \(S\) of \(n\) distinct keys, a function \(f\) that bijectively maps the keys of \(S\) into the range \(\{0, \ldots, n-1\}\) is called a minimal perfect hash function (MPHF) for \(S\).
- Lower bound of 1.44 bits/key - in practice: 2-4 bits/key and constant time evaluation.
- Many algorithms available:
- FCH [Fox et al., 1992]
- CHD [Belazzougui et al., 2009]
- EMPHF [Belazzougui et al., 2014]
- GOV [Genuzio et al., 2016]
- BBHash [Limasset et al., 2017]
- RecSplit [Esposito et al., 2019]
- PTHash [P. and Trani, 2021]


\section*{Sparse Hashing - Example}


\section*{Sparse Hashing - Example}


\section*{Skew Hashing - Example}

Example for \(\ell=3\).


\section*{Elias-Fano Encoding}
- Elias-Fano [Elias, 1974; Fano, 1971] is a succinct data structure representing a monotone integer list \(X[0 . . n)\) in \(n\left\lceil\log _{2}(U / n)\right\rceil+2 n\) bits, where \(U\) is such that \(U \geq X[n-1]\).
- With just \(+o(n)\) extra bits: random Access in \(O(1)\) and Predecessor queries in \(O(\log (U / n))\).
- Found to be crucial for many practical data structures/applications (e.g., inverted indexes, compressed tries, MPHF).
- See Section 3.4 of

Techniques for Inverted Index Compression
P. and Venturini, ACM Computing Surveys, 2021.
- https://github.com/jermp/data compression course

\section*{Skew Hashing}
- For \(i=\ell, \ldots, L\), let \(K_{i}\) is the set of all \(k\)-mers belonging to buckets of size \(s\), with \(s\) such that:
\[
\begin{cases}2^{i}<s \leq 2^{i+1} & \ell \leq i<L \\ 2^{L}<s \leq \max & i=L\end{cases}
\]
- We build a MPHF \(f_{i}\) for each set \(K_{i}\). For a \(k\)-mer \(g \in K_{i}\), we know that its bucket contains at most \(2^{i+1}\) super- \(k\)-mers, so we write the identifier of the super- \(k\)-mer containing \(g\) in a (compact) vector \(V_{i}\) of \((i+1)\)-bit ints.
- Upon Lookup, we will scan one super- \(k\)-mer only.

\section*{Trade-offs by Varying Minimizer Length}

Space in bits/k-mer (bpk) and Lookup time (indicated by \(\mathrm{Lkp}^{+}\)for positive queries; by \(\mathrm{Lkp}^{-}\)for negative) in average \(\mathrm{ns} / k\)-mer for regular and canonical SSHash dictionaries by varying minimizer length \(m\). For each dataset, we indicate promising configurations in bold font.
\begin{tabular}{|c|c|c|c|c|}
\hline \multirow{2}{*}{Dataset} & \(m\) & \(m\) & \(m\) & \(m\) \\
\hline & bpk Lkp \({ }^{+} \mathrm{Lkp}^{-}\) & bpk Lkp \({ }^{+} \mathrm{Lkp}^{-}\) & bpk \(\mathrm{Lkp}^{+} \mathrm{Lkp}^{-}\) & bpk \(\mathrm{Lkp}^{+} \mathrm{Lkp}^{-}\) \\
\hline Cod & 15 & 16 & 17 & 18 \\
\hline regular & 6.6012361267 & 6.8211001174 & 6.9810451158 & 7.2110151157 \\
\hline canonical & \(\begin{array}{llll}7.68 & 945 & 768\end{array}\) & \(\begin{array}{llll}7.92 & 834 & 690\end{array}\) & \(\begin{array}{lll}8.18 & 786 & 672\end{array}\) & \(8.47 \quad 755 \quad 658\) \\
\hline Kestrel & 16 & 17 & 18 & 19 \\
\hline regular canonical & \[
\begin{array}{rrr}
6.19 & 1137 & 1323 \\
\mathbf{7 . 3 0} & \mathbf{8 8 2} & \mathbf{7 8 1}
\end{array}
\] & \[
\begin{array}{rrr}
\mathbf{6 . 4 8} & \mathbf{1 0 4 2} & \mathbf{1 2 6 5} \\
7.68 & 790 & 722
\end{array}
\] & \[
\begin{array}{rrr}
6.79 & 1005 & 1245 \\
8.09 & 743 & 696
\end{array}
\] & \[
\begin{array}{rrr}
7.12 & 997 & 1240 \\
8.51 & 730 & 691
\end{array}
\] \\
\hline Human & 17 & 18 & 19 & 20 \\
\hline regular & 7.4415911668 & 7.6714591573 & 7.9514061547 & 8.2813381530 \\
\hline canonical & 8.761150936 & 9.041054881 & \(\begin{array}{lll}9.39 & 990 & 854\end{array}\) & \(\begin{array}{lll}9.80 & 958 & 838\end{array}\) \\
\hline Bacterial & 18 & 19 & 20 & 21 \\
\hline regular & 7.4215351867 & 7.8014251813 & 8.2213891780 & 8.7013681774 \\
\hline canonical & 8.7511291043 & 9.221051995 & \(9.751028 \quad 947\) & \(10.34 \quad 998 \quad 956\) \\
\hline
\end{tabular}

\section*{NOTE 1:}

We used \(l=6\) and \(L=12\) for all experiments.

\section*{NOTE 2:}

A good rule of thumb is
\(m=\left\lceil\log _{4}(N)\right\rceil+1\) or
\(m=\left\lceil\log _{4}(N)\right\rceil+2\).

\section*{Space Breakdowns}


Space breakdowns for the Human dataset, for both (a) regular and (b) canonical dictionaries. The numbers next to each bar indicate the bits/k-mer (bpk) spent by the respective components.

\section*{Construction Time and Space}

Dictionary construction times in minutes (using a single processing thread) and peak internal memory used during construction in GB. (Blight's performance was the same for all values of \(b\) in the experiment.)
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline \multirow{2}{*}{Dictionary} & \multicolumn{2}{|l|}{Cod} & \multicolumn{2}{|l|}{Kestrel} & \multicolumn{2}{|l|}{Human} & \multicolumn{2}{|l|}{Bacterial} \\
\hline & min & GB & min & & min & GB & min & GB \\
\hline dBG-FM, \(s=128\) & 28.5 & 0.5 & 100.0 & 0.7 & - & - & - & \\
\hline dBG-FM, \(s=64\) & 28.5 & 0.6 & 100.0 & & - & - & - & \\
\hline dBG-FM, \(s=32\) & 28.5 & 0.7 & 100.0 & 1.1 & - & - & - & - \\
\hline Pufferfish, sparse & 15.5 & 3.3 & 35.2 & 6.7 & 86.0 & 19.4 & 200.8 & 40.1 \\
\hline Pufferfish, dense & 13.0 & 2.8 & 29.2 & 5.9 & 70.7 & 14.0 & 173.2 & 30.4 \\
\hline Blight & 5.0 & 3.3 & 11.0 & 7.0 & 25.0 & 7.5 & 50.0 & 15.8 \\
\hline SSHash, regular & 1.5 & 2.6 & 3.8 & 5.7 & 12.5 & 15.4 & 29.6 & 33.4 \\
\hline SSHash, canonical & 2.0 & 2.8 & 4.4 & 5.8 & 16.2 & 17.3 & 36.0 & 36.6 \\
\hline
\end{tabular}

NOTE: SSHash construction works entirely in internal memory.
(This is going to change in future releases.)```

