# **Sparse and Skew Hashing of K-Mers\*** \*ISMB 2022, to appear.

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# **The K-Mer Dictionary Problem**

- K-Mer. A k-mer is a string of length k over the DNA alphabet {A,C,G,T}.
- We are given a large DNA string (e.g., a genome or a pan-genome) and let K be the set of all its *n* distinct *k*-mers.

Example: The human genome (GRCh38) has >2.5B distinct k-mers for k = 31.

- **Problem.** We want to build a dictionary for K so that the following operations are efficient: -i = Lookup(g), where  $0 \le i < n$  if the k-mer  $g \in K$  or i = -1 otherwise; - return the k-mer g = Access(i) if  $0 \le i < n$ .

(Other operations of interest are *iteration* and *streaming* membership queries. See the paper for details.)

## **K-Mer Applications**

- Software tools based on k-mers are predominant in Bioinformatics.
- Many applications:
  - genome assembly
  - variant calling
  - pan-genome analysis
  - meta-genomics
  - sequence comparison/alignment
  - ...

# de Bruijn Graphs

- Fact. Equivalence between a set of k-mers and a de Bruijn graph (dBG).
- There are efficient software tools to run the following pre-processing flow.



BCALM [Chikhi et al., 2016]
Cuttlefish [Khan and Patro, 2021]

- A collection of DNA strings with no duplicate k-mers.
- Efficient heuristic method to reduce the number of bases, e.g, UST
   [Rahman and Medvedev, 2020].

#### **Super-k-Mers**

the smallest sub-string of length  $m \leq k$  according to a given order R.

Example for k = 13 and m = 4: ACGGTAGAACCGATTCAAATTCGATCGATTAATTAGAGCGATAAC... ACGGTAGAACCGA CGGTAGAACCGAT GGTAGAACCGATT super-k-mer **GTAGAACCGATTC** TAGAACCGATTCA AGAACCGATTCAA GAACCGATTCAAA AACCGATTCAAAT

Super-k-mer. [Li et al., 2013] Given a string, a super-k-mer is a maximal sequence of consecutive k-mers having the same minimizer.

. . .

• **Property.** Consecutive k-mers are likely to have the same minimizer [Roberts et al., 2004] -

#### Super-k-Mers

- super-k-mers than k-mers approx. (k m + 2)/2 times less for random minimizers  $\rightarrow$  sparse indexing.
- Observation 2. A super-k-mer of length s is a space-efficient representation of the set of its long chains of super-k-mers, the cost becomes approx. 2 bits/k-mer.

Example for k = 13 and m = 4:

ACGGTAGAACCGATTCAAATTCGATCGATTAATT..

This **chain** is of length 31 and costs 2x31=62 bits for 19 k-mers (**3.26** bits/k-mer).

• Observation 1. Since consecutive k-mers are likely to have the same minimizers, there are far fewer

constituent s - k + 1 k-mers: 2s/(s - k + 1) vs. 2k bits/k-mer. If s is sufficiently large and/or we have



# **Sparse Hashing**

- Q. How to index super-k-mers?
- Do not break the chains of super-k-mers to avoid wasting 2(k-1) bits per super-k-mer.
- Locate super-k-mers with an array of offsets into the strings, indexed by a minimal perfect hash • function (MPHF) on the minimizers.
- Upon Lookup(g): if r is the minimizer of g, locate and scan the "bucket" of r the set of super-k-mers that have minimizer r.



TTTTCAGGTTCATCATCTCCCTTCTTTGCAGGATAGTAGATAAGATCGCTCATCAACGGATGTTGTGTAATTCTGGTAAGATGTTCTTCTAGATCATCCCAA TATTTGTCAAGCACTTCCCCTTTTAATTGAGCGTTATCCCCGG.AGATGATGAAGACCTGAAAACATCCTGAAAATTGTCAAAGAATGGCGGCGTTCACAGGGG 



#### **Skew Hashing**

#### • **Problem.** Some buckets can be very large. For example on the human genome (GRCh38), for k = 31 and m = 20: largest bucket size

can be as large as  $3.6 \times 10^4$ .

#### Property. Minimizers have a (very) skew distribution for sufficiently long length.

Bucket size distribution (%) for k = 31 and the first  $n = 10^9 k$ -mers of the human genome, by varying minimizer length *m*.

size / m	11	12	13	14	15	16	17	18
1	13.7	19.8	29.7	42.4	61.5	79.5	89.8	94.4
2	7.5	10.6	14.4	17.7	19.4	13.6	7.3	3.9
3	5.2	7.3	8.8	10.4	8.4	3.7	1.4	0.8
4	4.0	5.5	6.0	7.0	4.1	1.3	0.5	0.3
5	3.2	4.4	4.5	5.0	2.2	0.6	0.3	0.2

19	20	21
96.3	97.1	97.5
2.4	1.7	1.4
0.5	0.4	0.4
0.2	0.2	0.2
0.1	0.1	0.1

On the full human genome (GRCh38), for k = 31 and m = 20: 2,505,445,761 *k*-mers 421,845,806 minimizers 388,018,280 (91.98%) only appear once!

#### Skew Hashing

- We fix an integer  $\ell$ : by virtue of the skew distribution, the fraction of buckets having more than  $2^{\ell}$  super-k-mers is small.
- So, we can afford a MPHF over the set of k-mers that belong to such super-k-mers. The output of the MPHF for a k-mer g is the identifier of the super-k-mer where g is present. (This identifier can be written in few bits; see the paper for details.)
- Upon Lookup, we will scan **one** super-k-mer only.

Bucket size distribution (%) for k = 31 and the first  $n = 10^9 k$ -mers of the human genome, by varying minimizer length *m*.

size / m	11	12	13	14	15	16	17	18	19	20	21
1	13.7	19.8	29.7	42.4	61.5	79.5	89.8	94.4	96.3	97.1	97.5
2	7.5	10.6	14.4	17.7	19.4	13.6	7.3	3.9	2.4	1.7	1.4
3	5.2	7.3	8.8	10.4	8.4	3.7	1.4	0.8	0.5	0.4	0.4
4	4.0	5.5	6.0	7.0	4.1	1.3	0.5	0.3	0.2	0.2	0.2
5	3.2	4.4	4.5	5.0	2.2	0.6	0.3	0.2	0.1	0.1	0.1

For  $\ell = 2$ , just 100.0 - (97.1 + 1.7 + 0.4 + 0.2)% = 0.6% of buckets with more than  $2^{\ell=2} = 4$  super-k-mers.

# **Benchmarking SSHash**

- Code in C++17, compiled with flags: -03 -march=native.
- All experiments are **single-threaded**.
- We use  $\ell = 6$  for all experiments.
- We use m = 17, 17, 20, 20 for respectively Cod, Kestrel, Human, and Bacterial. (A good rule of thumb is  $m = \lceil \log_4(N) \rceil + 1$  or  $m = \lceil \log_4(N) \rceil + 2$ .)

Some basic statistics for the datasets used in the experiments, for k = 31, such as number of: k-mers (n), paths (p), and bases (N).

Dataset	n	p	N	$\lceil \log_2(N) \rceil$
Cod	502,465,200	2,406,681	574,665,630	30
Kestrel	1,150,399,205	682,344	1,170,869,525	31
Human	2,505,445,761	13,014,641	2,895,884,991	32
Bacterial	5,350,807,438	26,449,008	6,144,277,678	33

**NOTE**: We used BCALM (v2) [Chikhi et al., 2016] to build the compacted dBG and then UST [Rahman and Medvedev, 2020] to compute the stitched unitigs.

#### Competitors

- **dBG-FM** [Chikhi et al., 2014]: FM-index [Ferragina and Manzini, 2000]
- Pufferfish [Almodaresi et al., 2018]: MPHF
- **Blight** [Marchet et al., 2021]: MPHF+minimizers

#### **Overall Comparison – Space and Lookup**

#### Dictionary space in total GB and average bits/k-mer (bpk).

Dictionary	С	od	Ke	strel	Hui	nan	Bacterial		
Dictionary	GB	bpk	GB	bpk	GB	bpk	GB	bpk	
dBG-FM, $s = 128$	0.22	3.48	0.44	3.07	_	_	_	_	
dBG-FM, $s = 64$	0.27	4.38	0.55	3.86	_	_	_	_	
dBG-FM, $s = 32$	0.39	6.16	0.78	5.43	_	_	_	-	
Pufferfish, sparse	1.75	27.80	3.69	25.66	8.87	28.32	18.91	28.28	
	1.49	23.70	3.37	23.40	7.50	23.96	16.09	24.06	
Pufferfish, dense	2.69	42.76	5.97	41.54	14.11	45.04	30.70	45.89	
	2.43	38.66	5.65	39.28	12.74	40.68	27.88	41.68	
Blight, $b = 4$	0.91	14.53	2.16	15.00	5.04	16.11	11.40	17.04	
Blight, $b = 2$	1.04	16.57	2.45	17.04	5.67	18.13	12.74	19.05	
Blight, $b = 0$	1.17	18.61	2.74	19.06	6.32	20.17	14.12	21.11	
SSHash, regular	0.44	6.98	0.93	6.48	2.59	8.28	5.50	8.22	
SSHash, canonical	0.50	7.92	1.00	7.30	2.94	9.39	6.17	9.22	

- Compared to BWT-based indexes: one order of magnitude faster for "just" 2x more space.
- Compared to other hashing schemes: 2-5x smaller with comparable of faster query time.

Dictionary	Co	od	Kes	strel	Hur	nan	Bact	eria
Dietionary	Lkp <sup>+</sup>	Lkp <sup>-</sup>	Lkp <sup>+</sup>	Lkp <sup>-</sup>	Lkp <sup>+</sup>	Lkp <sup>-</sup>	Lkp <sup>+</sup>	Lk
dBG-FM, $s = 128$	22,980	16,501	23,934	16,764	_	_	_	
dBG-FM, $s = 64$	15,013	10,919	15,929	11,462	_	_	_	
dBG-FM, $s = 32$	11,386	7929	11,703	8073	_	_	_	
Pufferfish, sparse	1110	700	5456	769	13,656	862	27,748	9
Pufferfish, dense	624	439	635	485	720	519	816	5
Blight, $b = 4$	2520	2751	2743	3104	2820	3329	3105	39
Blight, $b = 2$	1800	1643	1916	1820	2008	1975	2095	21
Blight, $b = 0$	1571	1317	1692	1472	1780	1610	1859	17
SSHash, regular	1045	1158	1042	1265	1338	1530	1389	17
SSHash, canonical	834	690	882	781	990	854	1051	9

#### Dictionary Lookup time in average ns/k-mer.

#### **Overall Comparison — Streaming Queries**

Query time for streaming membership queries for various dictionaries. The query time is reported as total time in minutes (tot), and average ns/k-mer (avg). We also indicate the query file (SRR number) and the percentage of hits. Both high-hit (> 70% hits) and low-hit (< 1% hits) workloads are considered.

	С	od	Ke	strel	Hu	man	Bact	erial		С	od	Kestrel		Human		Bacter	
Dictionary	SRR12858649 81.37% hits		SRR11449743 74.60% hits		SRR5833294 91.65% hits		SRR5901135 87.79% hits		Dictionary	SRR11449743 0.659% hits		SRR12858649 0.484% hits		SRR5901135 0.002% hits		SRR583 0.086%	
	tot	avg	tot	avg	tot	avg	tot	avg		tot	avg	tot	avg	tot	avg	tot	
Pufferfish, sparse	0.6	214	14.1	609	17.0	651	9.1	691	Pufferfish, sparse	14.6	627	0.9	312	11.3	855	25.5	
Pufferfish, dense	0.2	92	8.5	368	10.5	402	5.3	404	Pufferfish, dense	8.7	374	0.2	92	5.8	435	13.6	
Blight, $b = 4$	2.1	766	32.5	1400	27.3	1041	11.4	864	Blight, $b = 4$	72.2	3112	6.6	2407	35.7	2704	253.2	
Blight, $b = 2$	1.2	453	16.6	714	17.5	670	8.6	648	Blight, $b = 2$	45.9	1978	3.0	1115	19.1	1445	117.7	
Blight, $b = 0$	0.8	282	10.8	464	11.5	440	5.8	434	Blight, $b = 0$	18.1	780	1.8	655	14.4	1088	32.2	
SSHash, regular	0.5	166	6.2	267	8.2	311	3.0	223	SSHash, regular	10.7	463	0.9	314	6.2	463	14.3	
SSHash, canonical	0.3	111	5.1	219	6.7	253	2.4	184	SSHash, canonical	5.1	220	0.4	155	2.5	183	6.4	

(a) high-hit workload

(b) low-hit workload



#### Conclusions

- SSHash is an efficient solution to the *K-Mer Dictionary problem*: good trade-off between space and time.
- (MPHF, <u>https://github.com/jermp/pthash</u>), Elias-Fano.
- Ingredients:
  - **Sparse indexing** to obtain good space effectiveness;
  - Skew hashing to guarantee fast lookup for "heavy" buckets.
- Code in C++17 is available at: <u>https://github.com/jermp/sshash</u>.

• Compared to BWT-based indexes: one order of magnitude faster for "just" 2X more space. Compared to other hashing schemes: 2-5X smaller with comparable of faster query time.

• Tool-box: spectrum-preserving string sets (SPSSs), minimizers, minimal perfect hashing



Thank you for the attention!

# **Preliminary Observations**

- The algorithmic literature about (compressed) string dictionaries is rich of solutions [Martínez-Prieto et al., 2016] (e.g., Front-Coding, path-decomposed tries, double-array tries), but are relevant for "generic strings":
  - variable-length,

. . .

- larger alphabets (e.g., ASCII),
- (usually) no particular properties of the strings to aid compression.
- Since k-mers are extracted consecutively from DNA, a k-mer following another one shares k-1bases (very low entropy).





- Example for k = 13.

# **A World of K-Mer Indexes**

- Huge research effort produced many types of indexes based on k-mers, with different:
  - representations (hashing, BWT-based, exact vs. approximate),
  - features (e.g., static vs. dynamic),
  - space/time trade-offs,
  - operations, ecc.
- Recent surveys on this topic:
  - Data Structures based on k-mers for Querying Large Collections of Sequencing Data Sets  $\bullet$ Marchet et al., Genome Research, 2020.
  - Data Structures to Represent a Set of k-long DNA Sequences Chikhi et al., ACM Computing Surveys, 2021.



#### de Bruijn Graphs

#### **Fact.** Equivalence between a set of k-mers and a de Bruijn graph (dBG).



(c) set of *stitched (maximal) unitigs* 

#### Minimizers

- Minimizer. [Roberts et al., 2004] Given a k-mer and an order relation R, the *minimizer* of length  $m \leq k$  is the smallest m-mer of the k-mer according to R.
- Example. Given g = ACGGTAGAACCGA



If *R* is the *lexicographic* order.

If R is defined by a random hash function h.

$$(k = 13)$$
 and  $m = 4$ :

```
h(GGTA) = 8265731 \leftarrow smallest hash code
    h(GAAC) = 91815379
     h(AACC) = 645793914
      h(ACCG) = 918417644
       h(CCGA) = 814188124
```

# **Minimal Perfect Hashing**

**MPHF.** Given a set S of n distinct keys, a function f that *bijectively* maps the keys of S into the range  $\{0, ..., n - 1\}$  is called a *minimal perfect hash function* (MPHF) for S.

- Lower bound of 1.44 bits/key in practice: 2-4 bits/key and constant time evaluation.
- Many algorithms available:
  - FCH [Fox et al., 1992]
  - CHD [Belazzougui et al., 2009]
  - EMPHF [Belazzougui et al., 2014]
  - GOV [Genuzio et al., 2016]
  - BBHash [Limasset et al., 2017]
  - RecSplit [Esposito et al., 2019]
  - PTHash [P. and Trani, 2021]

https://github.com/jermp/pthash



#### **Sparse Hashing – Example**

a collection of 4 stitched unitigs: 285 *k*-mers for k = 31, N = 408 bases

AGATGATGAACCTGAAAACATCCTGAAAATCGTCAAAGAATGGCGG TTTCAGGATGTTTTCAGGTTCATCATCTCCCTTCTTTGCAGGATAG TAGATAAGATCGCTCATCAACGGATGTTGTGTAATTCTGGTAAGAT GTTCTTCTAGATCATCCCAATATTTGTCAAGCACTTCCCCTTTTAA TTGAGCGTTATCCCCGG.AGATGATGAACCTGAAAACATCCTGAAA ATTGTCAAAGAATGGCGGCGTTCACAGGGGCTA.ATTGTCAAAGAA GTAGATAATAAAACTATATATGGAACATCATCGCATCTGG

24 minimizers, for m = 8

TCGTCAAA: 29 CATCCCAA: 172 ATCGTCAA: 20 GACTCTAA: 50 329 AACCTGAA: 0 246 ATCCTGAA: 9 255 GAACATCA: 364 GCAGGATA: 105 AGGGGCTA: 30 CTTGTTTA: 319 GAGCGTTA: 208 TTTAAAGA: 323 CTTCTAGA: 169 GGCTACCC: 33 CGTTATCC: 211 AGCACTTC: 189 AAGATCGC: 119 **AACTATAT: 353** CCTTCTTT: 97 TTCAGGTT: 89 ACGGATGT: 143 ACAGGGGT: 310 TGTCAAAG: 266 307 TAATTCTG: 157

offsets

# Sparse Hashing – Examp



p	le																
	(F	MPH PTHa	HF ash)														
J	f														(	comp	act vect
														1	of	[log <sub>2</sub>	$_2N$ ]-bit
14	15	16	17	18	19	20	21	22	23	24	25	27	28				
11	12	13	14	15	16	17	18	19	20	21	22	23	24	-			
30	319	208	323	169	33	211	189	119	353	97	89	143	310	266	307	157	
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	



#### **Skew Hashing — Example**

Example for  $\ell = 3$ .





#### **Elias-Fano Encoding**

- Elias-Fano [Elias, 1974; Fano, 1971] is a succinct data structure representing a monotone integer list X[0..n) in  $n \lceil \log_2(U/n) \rceil + 2n$  bits, where U is such that  $U \ge X[n-1]$ .
- With just +o(n) extra bits: random Access in O(1) and Predecessor queries in  $O(\log(U/n))$ .
- Found to be crucial for many practical data structures/applications (e.g., inverted indexes, compressed tries, MPHF).
- See Section 3.4 of Techniques for Inverted Index Compression P. and Venturini, ACM Computing Surveys, 2021.
- <u>https://github.com/jermp/data\_compression\_course</u>

#### **Skew Hashing**

• For  $i = \ell, \ldots, L$ , let  $K_i$  is the set of all k-mers belonging to buckets of size s, with *s* such that:

$$\begin{cases} 2^{i} < s \leq 2^{i+1} \\ 2^{L} < s \leq max \end{cases}$$

- We build a MPHF  $f_i$  for each set  $K_i$ . For a k-mer  $g \in K_i$ , we know that its bucket containing g in a (compact) vector  $V_i$  of (i + 1)-bit ints.
- Upon Lookup, we will scan **one** super-k-mer only.

 $\ell \le i < L$ i = L

contains at most  $2^{i+1}$  super-k-mers, so we write the identifier of the super-k-mer

#### **Trade-offs by Varying Minimizer Length**

Space in bits/k-mer (bpk) and Lookup time (indicated by  $Lkp^+$  for positive queries; by  $Lkp^-$  for negative) in average ns/k-mer for regular and canonical SSHash dictionaries by varying minimizer length *m*. For each dataset, we indicate promising configurations in bold font.

Dataset		т			т	m m				т				
Dutubet	bpk Lkp <sup>+</sup> Lkp <sup>-</sup>			bpk	bpk Lkp <sup>+</sup> Lkp <sup>-</sup>			bpk Lkp <sup>+</sup> Lkp <sup>-</sup>			bpk Lkp <sup>+</sup> Lkp <sup>-</sup>			
Cod		15			16			17			18			
regular	6.60	1236	1267	6.82	1100	1174	6.98	1045	1158	7.21	1015	1157		
canonical	7.68	945	768	7.92	834	690	8.18	786	672	8.47	755	658		
Kestrel		16			17			18			19			
regular	6.19	1137	1323	6.48	1042	1265	6.79	1005	1245	7.12	997	1240		
canonical	7.30	882	781	7.68	790	722	8.09	743	696	8.51	730	691		
Human		17			18			19			20			
regular	7.44	1591	1668	7.67	1459	1573	7.95	1406	1547	8.28	1338	1530		
canonical	8.76	1150	936	9.04	1054	881	9.39	990	854	9.80	958	838		
Bacterial		18			19			20			21			
regular	7.42	1535	1867	7.80	1425	1813	8.22	1389	1780	8.70	1368	1774		
canonical	8.75	1129	1043	9.22	1051	995	9.75	1028	947	10.34	998	956		

NOTE 1:

We used  $\ell = 6$  and L = 12 for all experiments.

#### NOTE 2:

A good rule of thumb is

- $m = \lceil \log_4(N) \rceil + 1 \text{ or }$
- $m = \lceil \log_4(N) \rceil + 2.$

#### **Space Breakdowns**



(a) regular, 8.28 total bpk

Space breakdowns for the Human dataset, for both (a) regular and (b) canonical dictionaries. The numbers next to each bar indicate the bits/k-mer (bpk) spent by the respective components.

- (b) canonical, 9.39 total bpk

#### **Construction Time and Space**

Dictionary construction times in minutes (using a single processing thread) and peak internal memory used during construction in GB. (Blight's performance was the same for all values of b in the experiment.)

Dictionary	Cod	Kestrel	Human	Bacterial		
	min GB	min GB	min GB	min GB		
dBG-FM, $s = 128$	28.5 0.5	100.0 0.7				
dBG-FM, $s = 64$	28.5 0.6	100.0 0.9				
dBG-FM, $s = 32$	28.5 0.7	100.0 1.1				
Pufferfish, sparse	15.5 3.3	35.2 6.7	86.0 19.4	200.8 40.1		
Pufferfish, dense	13.0 2.8	29.2 5.9	70.7 14.0	173.2 30.4		
Blight	5.0 3.3	11.0 7.0	25.0 7.5	50.0 15.8		
SSHash, regular	1.5 2.6	3.8 5.7	12.5 15.4	29.6 33.4		
SSHash, canonical	2.0 2.8	4.4 5.8	16.2 17.3	36.0 36.6		

NOTE: SSHash construction works entirely in internal memory. (This is going to change in future releases.)