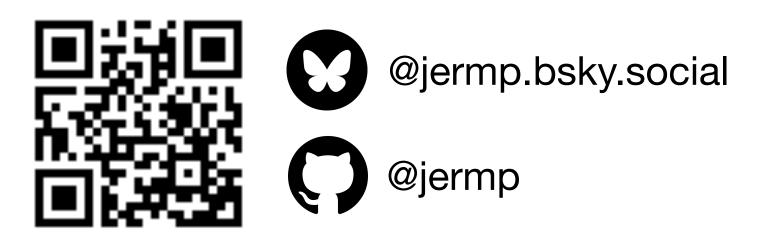
U-index: A Universal Indexing Framework for Matching Long Patterns

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23-rd Symposium on Experimental Algorithms (SEA 2025) Venice, Italy, 22 July 2025 Joint work with Lorraine A. K. Ayad Gabriele Fici Ragnar Groot Koerkamp Grigorios Loukides Rob Patro Solon P. Pissis

- Given a string T[0..n) over the alphabet $\Sigma = [0..\sigma)$, pre-process T so that the following queries can be answered efficiently for any string P[0..m):
 - Locate(P, T): return all the positions where P occurs in T;
 - Count(P, T): count the number of occurrences of P in T;
 - Extract(i, j, T): report the substring T[i ... j].
- Fundamental and well-studied problem.
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More about this on Friday: "25 years of compressed self-indexes" !

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 - **2. Uncompressed:** A redundancy (an "index") is attached to T to accelerate queries.



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 - **1. Compressed:** The text is replaced (is "self-indexed") with a compressed representation.

- Solutions in 1. are very space-efficient but generally slower to build and query than solutions in 2. which — on the other hand — are space-inefficient.
- **Example.** The (uncompressed) suffix array is much faster to query than the FM-index but requires $O(n \log n)$ bits on top of the text.



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Our contribution

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- be smaller/faster than Index(T) because S is a lot smaller than T, for any Index.
- At query time: we also compute Q = Sketch(P) and match Q against S. Candidate matches (including *false positives*) are mapped back to T to be verified.

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- At query time: we also compute Q = Sketch(P) and match Q against S. Candidate matches (including *false positives*) are mapped back to T to be verified.
- That is, we have a **universal framework** because:
 - any index can be used for S;

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- any locally-consistent sampling algorithm can be used to sketch the text and obtain S.

Intermezzo: sketching with minimizers

- and call it the "representative" of the window or its *minimizer*.
- We would like to sample the **same minimizer** from consecutive windows so that the **set of distinct minimizers** forms a succinct sketch for T.

• Consider each window of w consecutive k-mers from a string T: sample one k-mer out of w

Example for w = 4 and k = 7.

ACGGTAGAACCGATTCAAATTCGAT...

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Intermezzo: sketching with minimizers

Q. How do we compare different sampling algorithms?

(distinct) minimizers and the total number of k-mers of T.

The lower the density, the better!

A. We define the *density* of a sampling algorithm as the fraction between the number of

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Q. How do we compare different sampling algorithms?

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The lower the density, the better!

• Since the same k-mer cannot be a minimizer for more than w consecutive k-mers, we immediately have a **lower bound** of 1/w on the density of any sampling algorithm.



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The "folklore", random, minimizer

- 1: function MINIMIZER (W, w, k, \mathcal{O}_k) 2: $o_{min} = +\infty$ p = 03: for i = 0; i < w; i = i + 1 do 4: $o = \mathcal{O}_k(W[i..i+k))$ 5: if $o < o_{min}$ then 6: 7: $o_{min} = o$ 8: p = i9: return p
- We usually define the total order using a random hash function (random minimizer).
- In this case, the density is 2/(w + 1): almost a factor of 2 away from the lower bound for large w.

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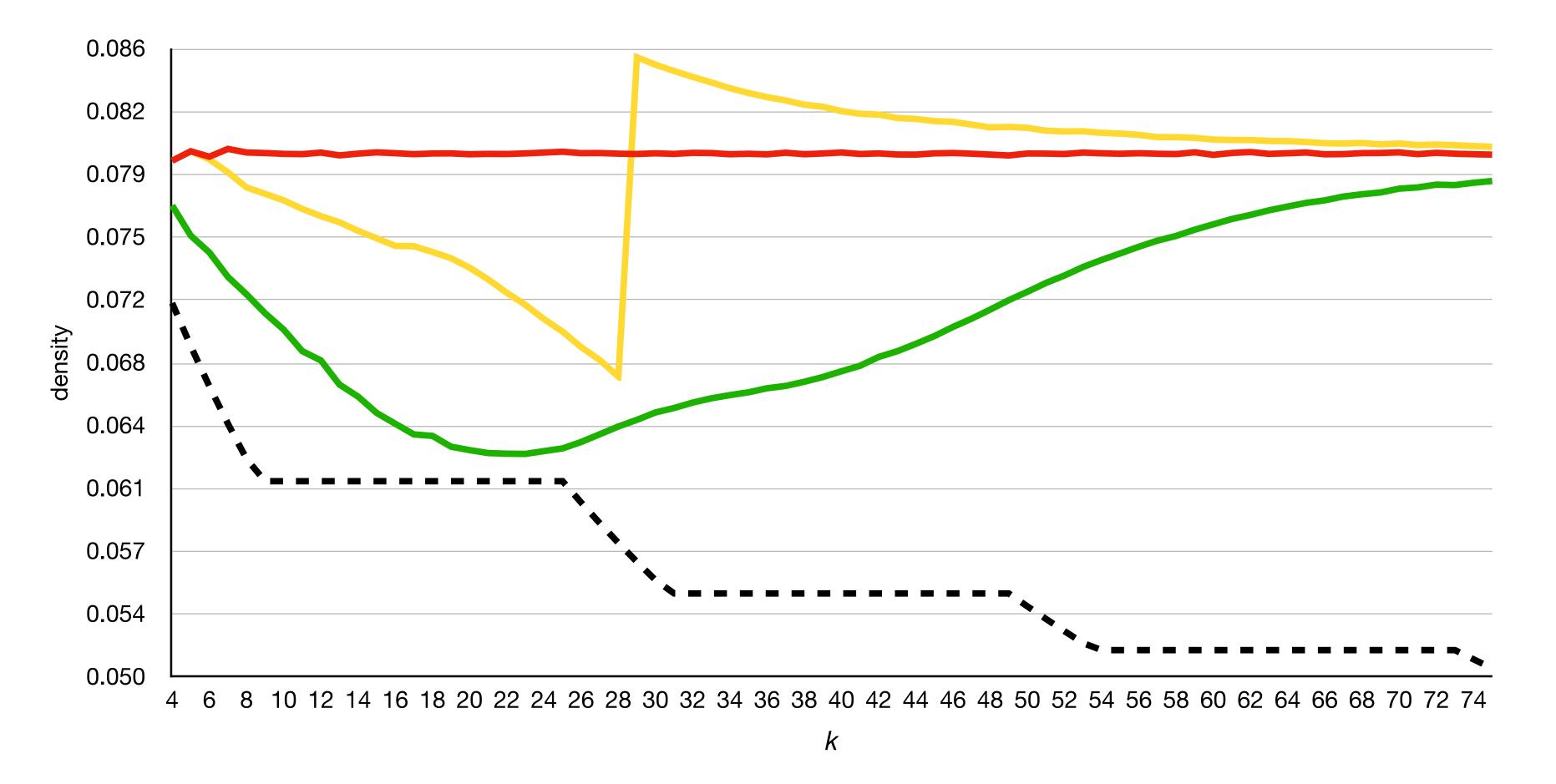
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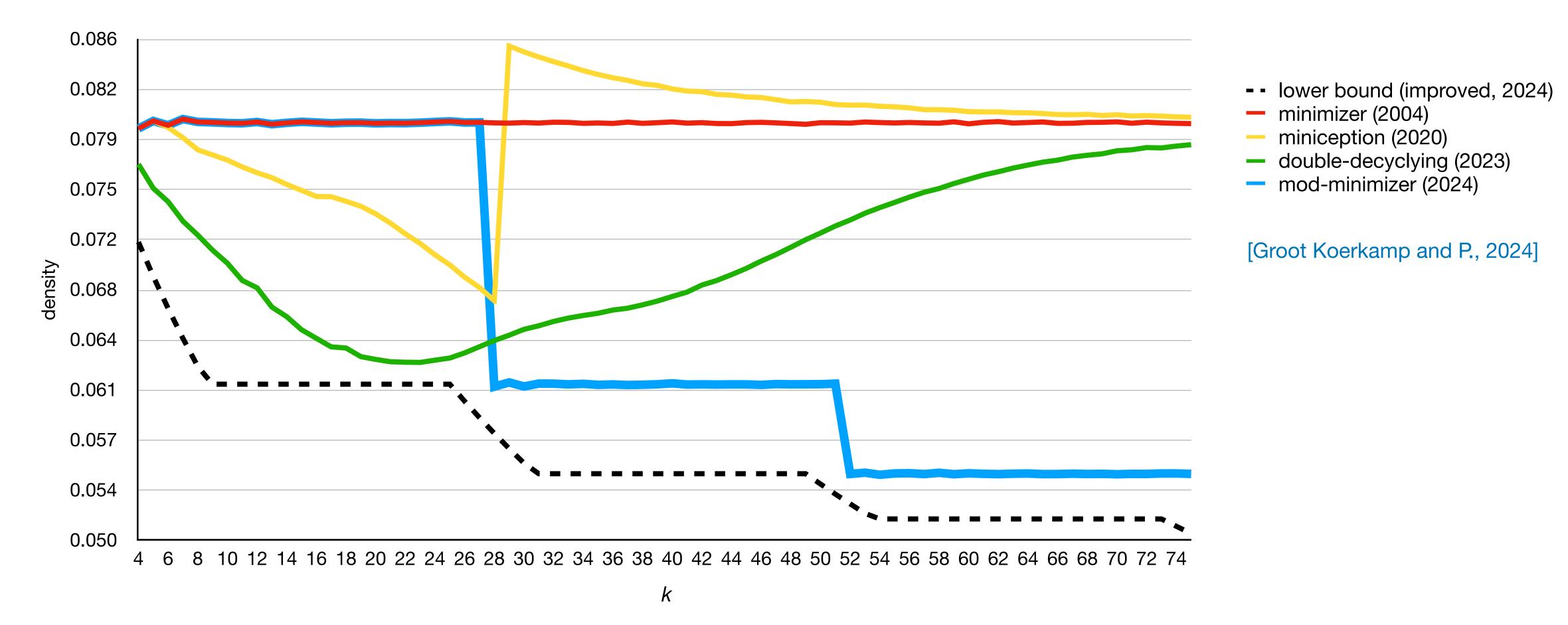
Density by varying k



- Example for w = 24. ullet
- Measured over a string of 10 million i.i.d. random characters with an alphabet size of 4. ullet
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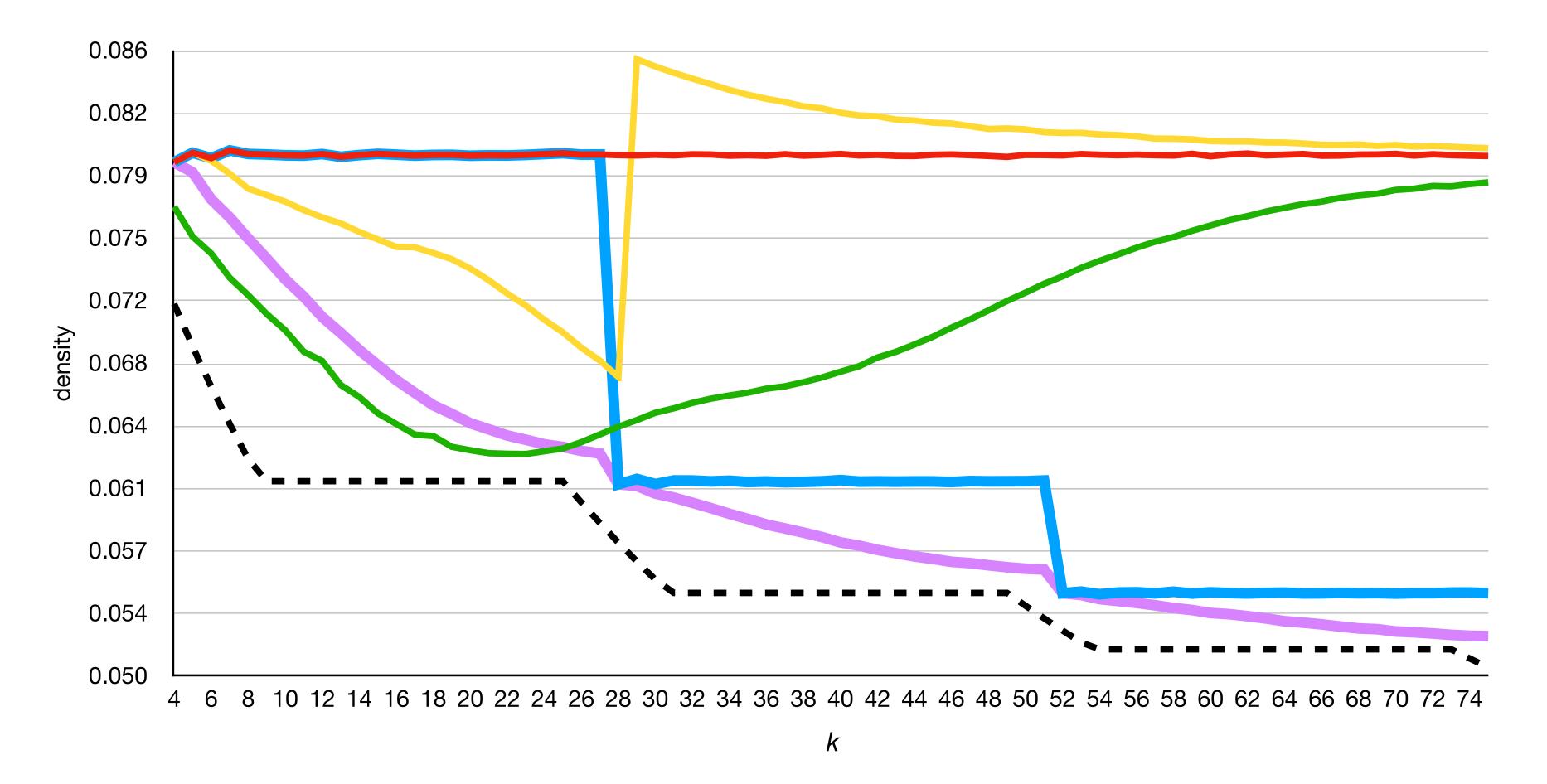
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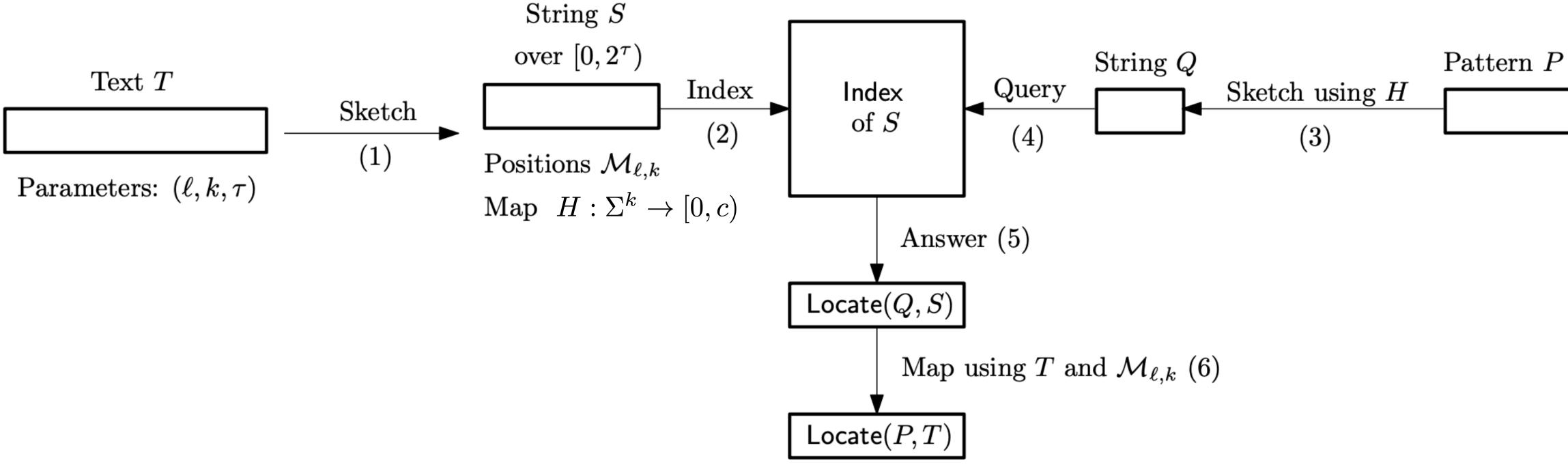
[Groot Koerkamp and P., 2024] [Groot Koerkamp, Liu, and P., 2025]





The U-index framework for matching long patterns

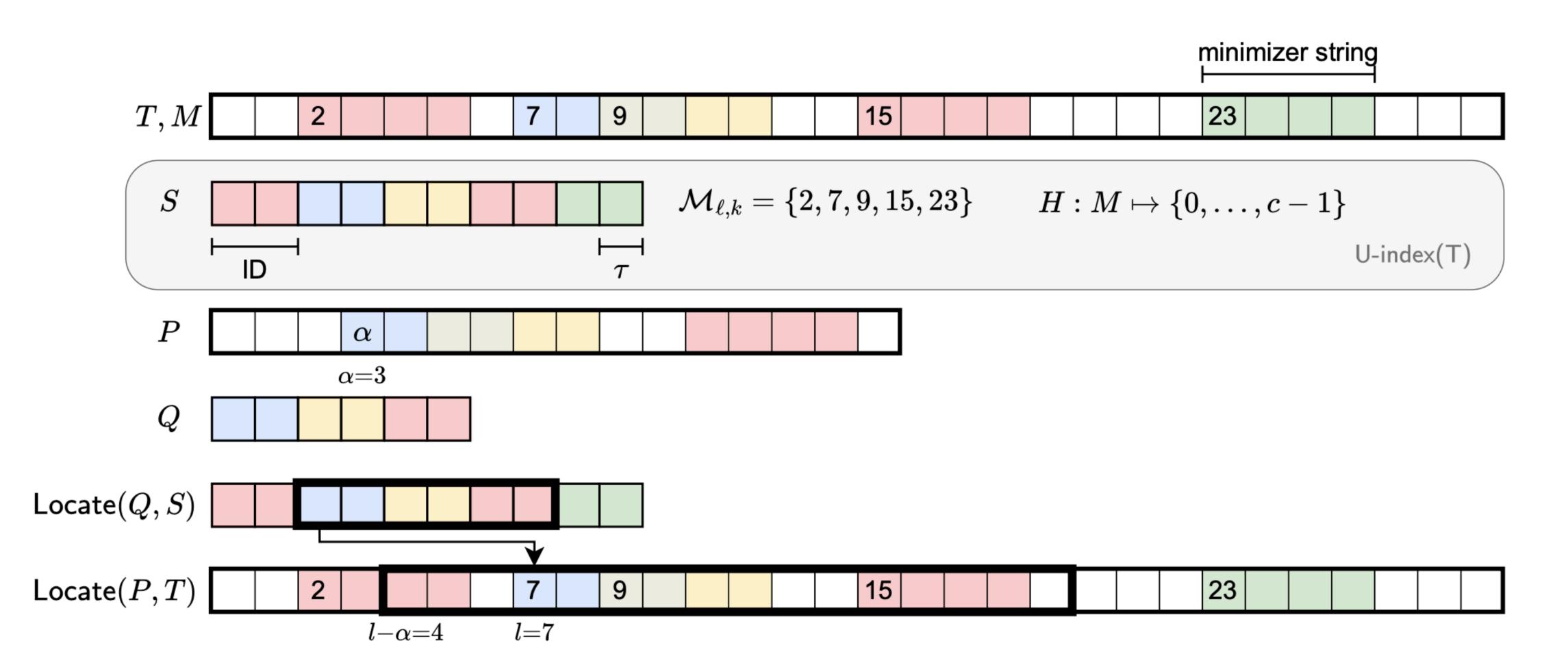
• $m \geq \ell$ contains at least one minimizer.



We fix integers $\ell > 0$ and k > 0 and let $w := \ell - k + 1$, so that any pattern P of length

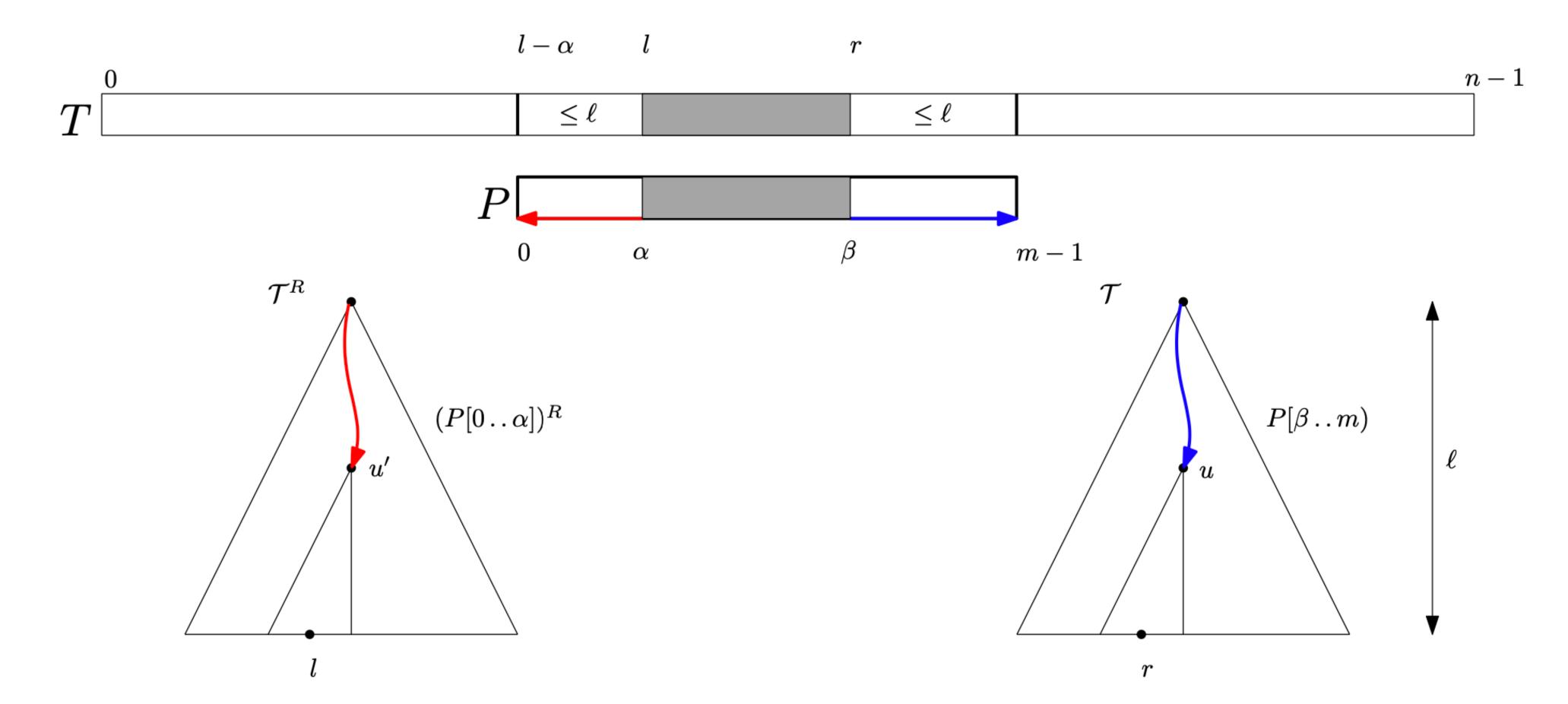


An example



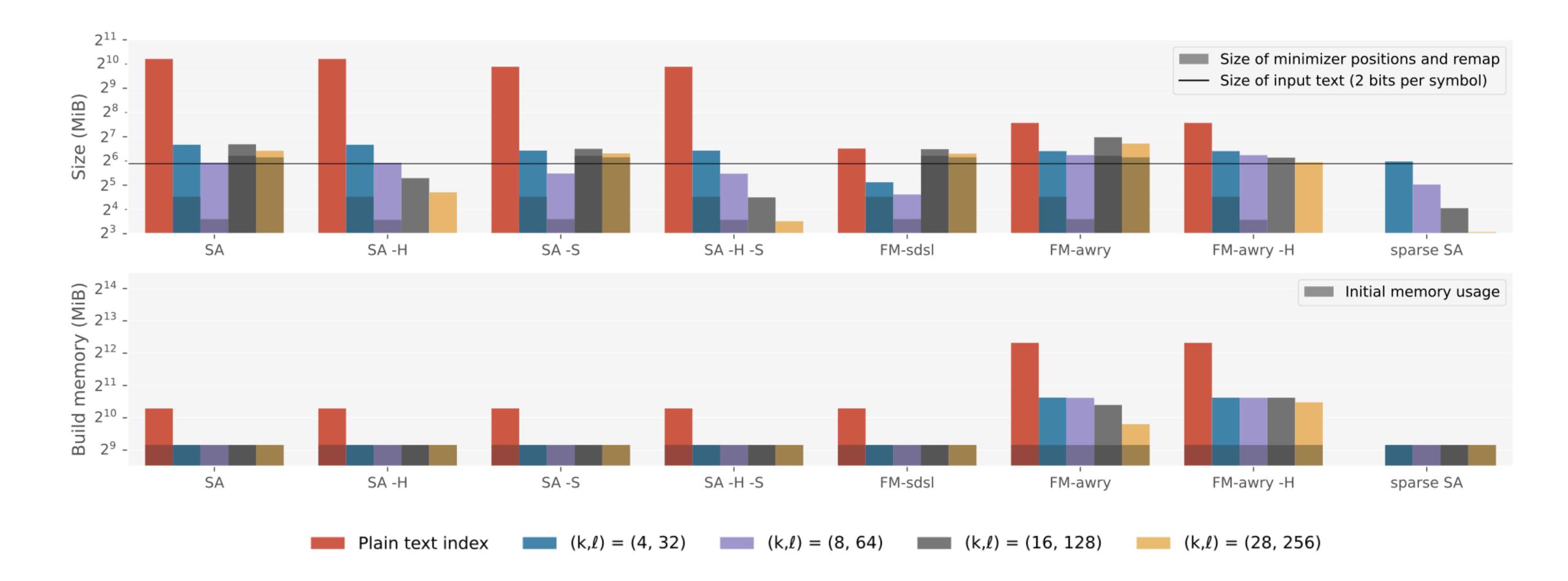
Theoretical guarantees

lacksquarenumber of minimizers of T.

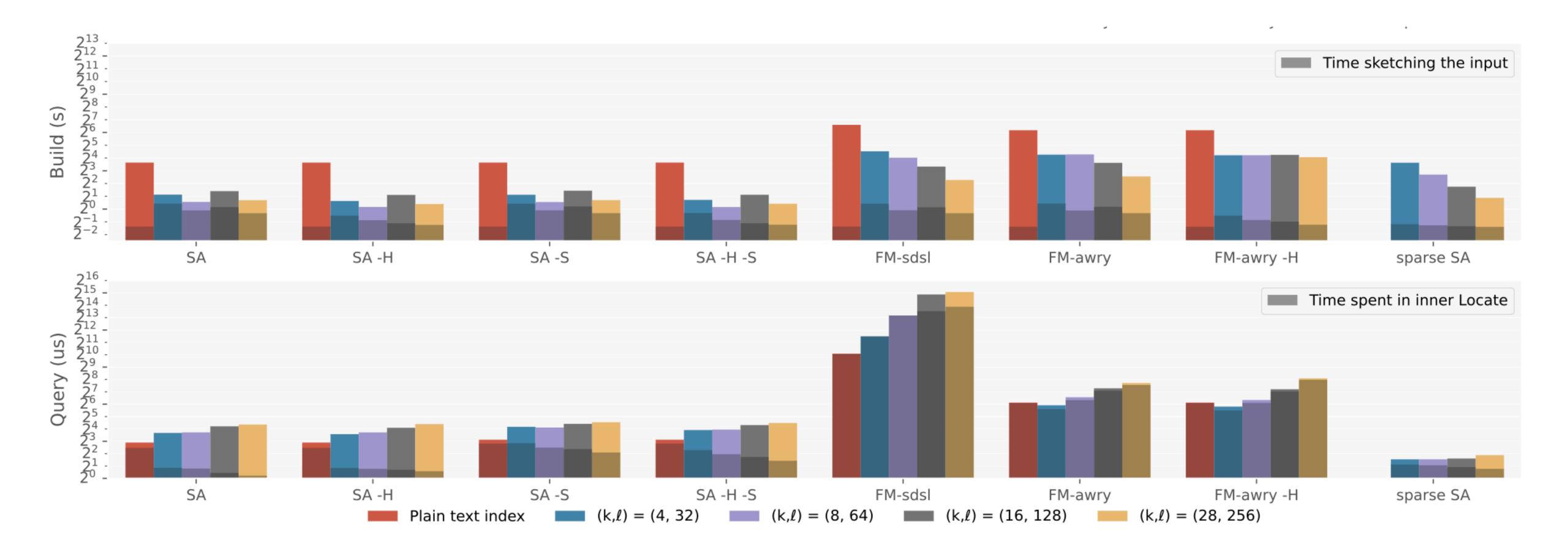


Using some machinery, we guarantee that an occurrence $p \in Locate(Q, S)$ is verified in O(1), rather than O(m). This can be done in O(z) space on top of the space of the text, where z is the

Results — Index size and build space for human chr 1



Results — Query and build time for human chr 1

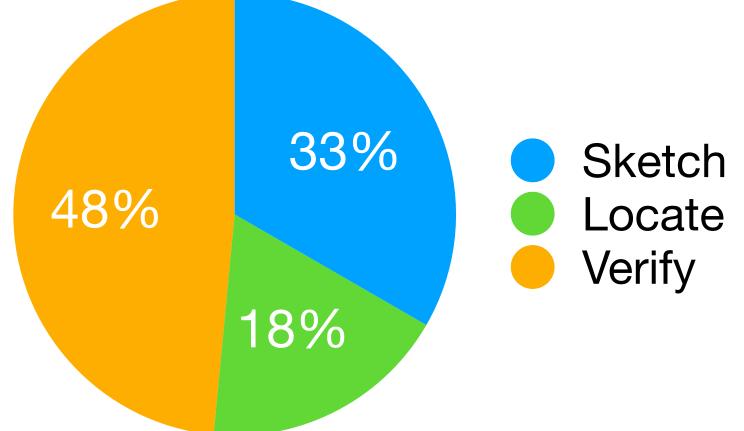


An example application

- We demonstrated that the U-index framework can be useful for long read mapping.
- A core problem in Computational Biology; it involves aligning long patterns to a reference genome.
- Experimental setting: we align 450 HiFi long reads (avg. length is 16 kbp) on a complete human reference genome. We use k = 8 and $\ell = 128$.

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- Very practical numbers **using a suffix array** as index: the U-index is built in 12 seconds with $\approx 9\mu$ s per pattern (23 avg. false positives per pattern).



Conclusions

- text index, provided that the patterns to match are reasonably long.
- index and sampling scheme used.
- Bottleneck: verifying false positive matches.
- Rust code: <u>https://github.com/u-index/u-index-rs</u>

Main take-away: U-index is a framework to enhance the performance of any off-the-shelf

The framework is very flexible: many space vs. time trade-offs possible depending on the

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Thank you for the attention! A special thank to all my co-authors!

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